Contagion and Systematic Risk:

an Application to the Survival of Hedge Funds

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ABSTRACT

This paper explores the modeling and measurement challenges of systematic risks and contagion for failure events, with an application to hedge funds' survival. The dependence in individual liquidation risks results either from an exogenous common factor with joint effects on the survival intensities, or from contagion phenomena which make the intensities dependent on past liquidations. In order to get tractable models for estimation and prediction purposes, we perform the analysis at a semi-aggregate level and consider the liquidation counts of several management styles. We introduce a dynamic model for multivariate count data with both lagged count values (contagion) and unobserved factors (dynamic frailty) among the regressors. The assumptions ensure that the joint process of liquidation counts and common factor is affine to facilitate nonlinear prediction at any horizon and estimation by a new method of moments. Our empirical analysis shows that the common factor, the sensitivities to this factor and the contagion scheme can be interpreted in terms of liquidity risks. The factor is related nonlinearly to rollover and margin funding liquidity risks. The sensitivities to the factor are funding liquidity risk exposures, which depend on the redemption and leverage policies of fund managers. The causal scheme captures the reinforcing spiral between funding and market liquidity risks.

Keywords: Systemic Risk, Contagion, Stress-Tests, Dynamic Count Model, Hedge Fund, Funding Liquidity.

JEL classification: G12, C23.

1 Introduction

This paper explores conceptual modeling and measurement challenges of systematic risks and contagion in Finance. This is illustrated by an application to hedge funds' survival with interpretations in terms of funding and market liquidity risks. This topic is especially relevant for the new regulation concerning financial stability. This regulation suggests to rank the financial institutions (including banks, credit institutions, insurance companies and possibly funds) according to their levels of systematic risks, to highlight the so-called Systematically Important Financial Institutions [SIFI's; see e.g. Financial Stability Board (2013)] [1], and to fix the reserves to hedge these risks.

A large part of the literature on financial contagion and systematic risks has been motivated by the finding that cross-market correlations (resp. coexceedances) between asset returns increase significantly during crisis periods; see e.g. King, Wadhwani (1990) [resp. Bae, Karolyi, Stulz (2003)]. Is this increase due to a shock common to all markets (called interdependence in the literature), or due to certain types of transmission of shocks between markets (called contagion)?

The starting point of our work is that it is not possible to identify the source of these increased co-movements in a purely static framework. Indeed, a static multivariate model for risk variables, such as returns, admit observationally equivalent representations in terms of either a simultaneous equation model, or recursive forms, without the possibility to interpret them in a structural way, for instance to interpret a recursive form as a causal relationship [2]. This is the reflection problem highlighted by Manski (1993).

To circumvent this difficulty some authors introduced multivariate models with exogenous switch-

[1]There are discussions among professionals and academics on the definitions and differences between systemic and systematic risks [see e.g. Hansen (2013)]. This difficulty appears clearly in the terminology SIFI, where the first S was originally meaning "Systemically" and became "Systematically" recently.

[2]A random vector Y with mean zero (for expository purpose) and variance-covariance matrix Σ, say, can be represented as the solution of a simultaneous equation system $Y = CY + u$. The matrix of coefficients C and the diagonal variance-covariance matrix Ω of the zero-mean error term u are such that $\Sigma = (Id - C)^{-1}\Omega(Id - C')^{-1}$. Since the matrix square root is not unique, matrices C and Ω are not identifiable. In particular, there exist representations with a triangular matrix C, corresponding to recursive systems. Due to the identification problem, they cannot be interpreted as causal representations.

ing regimes [see e.g. Forbes, Rigobon (2002), Dungey et al. (2005)]. These models are static in each regime, that are the tranquility regime and the crisis regime, respectively. The basic idea is to allow for a common factor and no contagion in the tranquility regime, whereas the crisis regime can involve contagion and/or extreme values of the common factor. Then, it is possible to identify the main source of increased co-movement by performing standard Chow tests. However, this solution to the identification problem requires identification restrictions, such as the fact that "the common shocks and the idiosyncratic shocks have the same impact during the crisis period as they have during the non-crisis period" [Dungey et al. (2005), p.11]. These identification restrictions are not necessarily fulfilled empirically and are not testable. Typically the identification problem will reappear, if contagion exists also in the tranquility regime, just at a less extent than in the crisis regime. The discussion above shows that we can expect to identify common factors and contagion only in a dynamic framework. The study of such a framework is the purpose of our paper.

To disentangle the effects of common shocks and contagion, a dynamic model requires at least three characteristics: first, the model has to include lagged endogenous variables to represent the propagation mechanism of contagion; second, the specification has to allow for two different sets of factors, namely common factors - also called either systematic factors, global factors [Karolyi, Stulz(1996)], or dynamic frailties [Duffie et al. (2009)] - representing undiversifiable risks, and idiosyncratic factors representing diversifiable risks; finally, these factors have to satisfy exogeneity properties. [3] These exogeneity properties are required for a meaningful definition of shocks on the factors. If a factor is exogenous, a shock on this factor is external to the system and cannot be partially a consequence of contagion. For instance, in a linear dynamic framework, a model disentangling contagion and common

[3] Even if both common factors and lagged endogenous variables have to be introduced in the model to get an appropriate interpretation of exogenous systematic risk and contagion, the literature often neglects one component, with the risk of misleading interpretations. For instance, Ait-Sahalia, Cacho-Diaz, Laeven (2010) and Billio et al. (2012) consider contagion models without exogenous common factor, whereas Boyson, Stahel, Stulz (2010), Section II.B, use a model with observable factors only.

factors might be:

$$Y_t = BF_t + CY_{t-1} + u_t, \qquad \text{with } F_t = AF_{t-1} + v_t, \tag{1.1}$$

where Y_t is the vector of endogenous variables, (u_t) and (v_t) are independent strong white noises, and the components of vector u_t are mutually independent. Vector F_t collects the values at time t of the common factors, and the components of vector u_t are the idiosyncratic factors. The exogeneity of the common factors is implied by the independence assumption between the errors u_t and v_t. Matrix C summarizes the contagion effects. Under standard stability conditions the dynamic model (1.1) admits a long run equilibrium given by the following static model:

$$Y = BF + CY + u, \quad \text{with} \quad F = AF + v,$$

or equivalently,

$$Y = (Id - C)^{-1}BF + (Id - C)^{-1}u, \quad \text{with} \quad F = (Id - A)^{-1}v. \tag{1.2}$$

We can now understand why a static model is hopeless for analyzing contagion. Indeed, (1.2) corresponds to the long run equilibrium model, provides no information on the tatonnement to converge to the equilibrium, and does not allow to identify the contagion matrix C.

It is usual to consider the idiosyncratic risk factors, i.e. the components of vector u_t in (1.1), as unobservable error terms, but the common risk factors are often assumed observable in the literature. For instance, in their analysis of hedge fund contagion based on returns, Boyson, Stahel, Stulz (2010), Section II.C, consider a model with observable factors, lagged "worst" return for the within-class contagion effects and contemporaneous "worst" return for the between-classes contagion effect. The use of observable factors facilitates the estimation, which can disregard the factor dynamics and is feasible even if the model includes a large number of factors. But this practice has to be avoided in contagion analysis for the following reasons: *i)* The standard literature never checks the exogeneity of the observable factors. When they are not exogenous, the estimated contagion matrix is biased; *ii)* It is necessary to estimate also the common factor dynamics in order to predict the future risks. The estimation of such common factor dynamics is a difficult task when the model involves many explanatory factors; *iii)* The assumption of latent common factors is mandatory in the new financial regulation to capture the uncertainty on the sources of risk correlation, and take it into account when computing the required capitals or pricing derivatives.

Our paper introduces a nonlinear dynamic model with both unobservable common factors and contagion for the analysis of the liquidation count histories of hedge funds (HF) in different management styles. We investigate empirically two causes of liquidation risk dependencies in the hedge fund industry.

i) The first one is the dependence due to frailty effects. There exist underlying exogenous stochastic factors with common influence on the liquidation intensities of the individual HF. For instance, the liquidation dependence may be due to the capital withdrawal of some prime brokers, and can be amplified by the use of debt to create the needed leverage. If the prime brokers simultaneously quit several funds, we observe a frailty effect, that is, a common risk factor effect. This frailty effect is exogenous, even when there is a herding behaviour of prime brokers, as long as their decision is not triggered by past liquidation events. It is an example of funding liquidity risk.

ii) The second cause of liquidation risk dependence is contagion. Indeed, the risk dependency can also arise when a shock specific to one fund has an impact on the probability of liquidation of other funds. For HFs this effect corresponds mainly to the market liquidity risk. If HF portfolios are invested in illiquid assets, it can be difficult and risky to continue to manage funds during a market liquidity crisis. Indeed, the fire sales of a given fund manager will consume the market liquidity of a given class of illiquid assets. The first consequence of such fire sales is a price pressure on these assets, which implies a decrease of the market value of all funds holding also these assets in their portfolio. This effect concerns the asset component of the balance sheet and is often called contagion in the HF literature. A well-known example is the default of the Russian sovereign debt in August 1998, when Long Term Capital Management (LTCM) and many other fixed-income HF suffered catastrophic losses over the course of a few weeks. Then, the failure of one of such funds increases the probability of liquidation of other funds. Another example is the increase of margin calls for hedge funds with large exposure in subprime-related fixed income securities, which forced them to sell securities held in their portfolios during the recent financial crisis. [4]

To the best of our knowledge, this is the first paper to introduce both frailty and contagion effects in HF survival models, and to measure the magnitudes of these effects. The liquidation intensity of an individual HF is assumed to depend on the lagged observations on liquidation counts in the same management style, or in other styles (contagion), as well as on a common unobservable dynamic factor (frailty). The specification allows to disentangle the two types of liquidation risk dependence by exploiting the time lag that contagion necessitates to produce its effects. The model is applied to the liquidation counts of the management styles. This semi-aggregation allows us to focus on the underlying systematic dynamic factors as well as on the contagion within and between management styles, as the hedge fund specific aspects of the liquidation process become negligible. The liquidation counts per management style are assumed to follow an autoregressive Poisson model with both frailty and contagion effects, which is the counterpart of model (1.1) for count data. We develop the estimation methods for this nonlinear dynamic model and the filtering algorithm to recover the values of the underlying unobservable factor.

The first result of our empirical analysis on the Lipper TASS database is that the shared dynamic

[4]There exist other reasons for liquidating HF. The income of a fund manager depends on the management fees, which are indexed in a complicated way on the performance of the fund, as well as on the total Asset Under Management (AUM). Therefore, when the outflows of capital are too large, the manager may decide to close the fund. Moreover, the effect of capital outflow is amplified by the specific high water mark fee structure of the HF manager [see e.g. Brown, Goetzmann, Liang (2004), Darolles, Gourieroux (2014) for the description of fees].

frailty is the reason of the major part of the liquidation clustering in a portfolio of hedge funds. The direct effect of contagion, that is, the transmission of the idiosyncratic shock to an individual HF within and between management styles, is rather limited. However, the contagion scheme has a quantitatively important indirect effect through the amplification of the shocks to the shared frailty. The second empirical finding is that the common factor, the sensitivities of the liquidation counts to this factor, and the contagion scheme have all interpretations in terms of liquidity risks. The underlying common factor provides a measure of rollover and margin funding liquidity risks with two regimes. The sensitivities to the factor provide the liquidity risk exposures of the different management styles; they are linked to the redemption frequency and management of gates by the HF managers. Finally, the estimated causal scheme of contagion captures a part of the spiral between funding and market liquidity risks highlighted in Brunnermeier, Pedersen (2009).

The understanding of the systematic patterns of dependence between the individual liquidation risks in the hedge fund industry is crucial for financial market regulation. The supervisors have to monitor both the funding and market liquidity risks. They may modify and control the funding liquidity exposure by means of restrictions on the use of leverage, the redemption frequency and the minimal requirements for investing in a HF. Supervisors may limit the market liquidity exposure by applying the Basel III regulations, for instance by introducing additional reserves based on liquidity stress scenarios. The Poisson model with frailty and contagion is especially appealing to analyze the consequences of stress on either funding, or market liquidity in a dynamic framework. It allows for designing the policies that would attenuate their consequences. In this paper we illustrate how our model can be used to *i)* predict the distributional properties (e.g. the mean and the quantiles) of the liquidation rate in a given HF management style at any horizon, and *ii)* study the effects of stress scenarios on these predicted liquidation rates. In particular, the stress on the current value of the frailty (shocks on systematic funding liquidity risk) and the stress on the contagion matrix (shocks on the speed of propagation) can be accommodated. The analysis of the common risk factors and contagion effects is the first step in the assessment of the impact of the HF industry on systemic risk for the global financial markets and the possibility of cascades into a global financial crisis.

The paper is organized as follows. In Section 2 we introduce the Poisson contagion model with dynamic frailty. The model with autoregressive gamma frailty is especially convenient, since it provides a joint affine dynamics for the frailty and the liquidation counts. This facilitates the prediction of future liquidation risk as well as the estimation of parameters in such a nonlinear setting with unobservable factors. In Section 3 we describe the dataset on hedge fund liquidation used in our empirical study. We aggregate the liquidation counts per management style. The dynamic model with contagion and frailty is estimated in Section 4. We assess the relative magnitude of contagion and shared frailty phenomena when we study liquidation risks dependence across different management styles. We carefully distinguish between the direct frailty effect and the amplification of the exogenous systematic shocks through the contagion network. In Section 5 we discuss the interpretations of the estimated causal scheme and factor sensitivities in terms of liquidity risks. We also derive the filtered values of the underlying unobservable factor and show that the filtered path is related in a nonlinear way with the standard proxies of the rollover and margin funding liquidity risks. We discuss the observed nonlinearity in terms of endogenous switching regimes. In Section 6 we illustrate our methodology by an application to dynamic stress tests of HF portfolios, that evaluates the stress effects on the term structure of liquidation risk. We perform different stress analyses by introducing shocks to either the systematic factor, its dynamics, or the magnitude of the contagion in the spirit of the new regulations for financial stability. Section 7 concludes. Technical proofs are gathered in the appendices and online supplementary material.

2 A multivariate dynamic Poisson model with frailty and contagion

In this section we introduce a dynamic model for the joint distribution of the liquidation count histories of hedge funds in different management styles. The model disentangles the fundamental exogenous common shocks (frailty) from the propagation of such shocks by the contagion phenomenons within and between management styles.

In each month t and each management style $k = 1, ..., K$, we observe the number $n_{k,t}$ of HF alive at the beginning of the month, and the liquidation count $Y_{k,t}$ during the month. The value of the unobservable common factor for month t is denoted by F_t. We specify the model in two steps. First, we specify the conditional distribution of the vector $Y_t = (Y_{1,t}, ..., Y_{K,t})'$ of liquidation counts given the current and past factor values $\underline{F_t} = (F_t, F_{t-1}, ...)$ and the past liquidation counts $\underline{Y_{t-1}} = (Y_{t-1}, Y_{t-2}, ...)$. Second, we specify the conditional distribution of the factor value F_t given the past histories $\underline{F_{t-1}}$ and $\underline{Y_{t-1}}$. These two conditional distributions are given in Assumptions A.1 and A.2, respectively, which are introduced in the next two subsections.

2.1 The conditional distribution of the liquidation counts

Assumption A.1: *The liquidation counts $Y_{k,t}$ are conditionally independent across management styles $k = 1, ..., K$ given $\underline{F_t}$ and $\underline{Y_{t-1}}$, with Poisson distribution:*

$$Y_{k,t}|\underline{F_t}, \underline{Y_{t-1}} \sim \mathcal{P}\left[\gamma_{k,t}(a_k + b_k F_t + c_k' Y_{t-1}^*)\right], \quad \textit{independent across } k = 1, ..., K, \tag{2.1}$$

where $Y_t^* = (Y_{1,t}/n_{1,t}, ..., Y_{K,t}/n_{K,t})'$ is the vector of liquidation rates, a_k and b_k are scalar coefficients, c_k is a vector of coefficients of dimension K, and $\gamma_{k,t} = n_{k,t}/n_{k,t_0}$.

This specification is inspired by the literature in epidemiology on contagion [see e.g. Anderson, Britton (2000)]. This literature goes back to the work of Sir Ronald Ross [Ross (1911)], who was awarded the Nobel Prize in Medicine in 1902, and of his students Kermack and McKendrick [Kermack and McKendrick (1927, 1932, 1933)]. The baseline intensity includes two components. The first one, $a_k + b_k F_t$, is the intensity of getting the disease via the exogenous factor represented by F_t and the vector $b = (b_1, ..., b_K)'$ of exposures to this factor. For instance, in the case of the Asian flu, the common factor is the contact with birds. This factor is exogenous, since there is no contagion from humans to birds. In analogy with Duffie et al. (2009) in an application to credit risk, we refer to the unobservable common factor F_t as the "dynamic frailty". [5] The second component, $c_k' Y_{t-1}^* = \sum_{l=1}^{K} c_{k,l} Y_{l,t-1}^*$, is the analogue of the intensity to get the disease via the contact with a sick human, in the same or in a different subpopulation. The contagion is introduced with a lag effect to capture the propagation phenomenon, which is inherently dynamic. [6] It is also necessary to adjust the model *i)* for the time-varying sizes of the subpopulations, via the term $\gamma_{k,t} = n_{k,t}/n_{k,t_0}$ scaling the baseline intensity, and *ii)* for the density of sick people in the subpopulations, via the use of Y_{t-1}^* instead of Y_{t-1} as explanatory variable. To ensure positivity of the liquidation intensity, we assume that the frailty process F_t is positive valued, and that the model coefficients a_k, b_k and $c_{k,l}$ are non-negative.

The contagion effect is measured by means of the $K \times K$ contagion matrix C with rows c_k', $k = 1, ..., K$. This modeling enables to account for contagion within as well as between management styles, since both diagonal and nondiagonal elements of the contagion matrix C can be nonzero.

[5] The notion of frailty has been initially introduced in duration models in Vaupel, Manton, Stallard (1979) and later used to define the Archimedean copulas [Oakes (1989)]. In this meaning, the frailty is an unobservable individual variable introduced to account for omitted (time-independent) individual characteristics and correct for the so-called mover-stayer phenomenon [see e.g. Baba, Goko (2006) for the introduction of an individual static frailty in the literature on HF survival]. In our framework, F_t is indexed by time and common to all HF, which justifies the terminology "dynamic frailty".

[6] It is often believed that there is "no consensus on what constitutes contagion and how it should be defined" [Forbes, Rigobon (2002); see also Bekaert, Harvey, Ng (2005)]. This lack of accordance on what contagion means is likely due to the fact that the financial literature has mostly focused on the "contagion" between asset returns. Indeed, when the analysis concerns asset returns, the interpretation suggested by the epidemiological models is not applicable. Following Eichengreen, Rose, Wyplosz (1996), some authors including Bae, Karolyi, Stulz (2003) and Boyson, Stahel, Stulz (2010) circumvent this difficulty by replacing the analysis of asset returns by the analysis of "risk infected" assets. An asset (hedge fund, or stock) is "risk infected", i.e. "sick" in epidemiological terms, if its return is below some cutoff point, such as the 10% quantile of the overall return distribution.

As discussed in the Introduction, for HF liquidation the intensity specification (2.1) admits an economic interpretation that differs from the one in epidemiological models. The exogenous shocks are mainly shocks to liability due to large cash withdrawals of investors. This is the redemption risk discussed in Diamond, Dybvig (1983), and Shleifer, Vishny (1997). The contagion due to fire sales goes through the asset component of the balance sheet. "Market liquidity and funding liquidity are mutually reinforcing leading to a liquidity spiral" [Brunnermeier, Pedersen (2009)].

The choice of a conditional Poisson distribution for the liquidation counts in (2.1) can be motivated as follows. Suppose that there exist a number of microscopic contagion models for the individual hedge funds. In each microscopic model, the liquidation intensity of a fund in management style k at month t is proportional to $a_k + b_k F_t + c_k' Y_{t-1}^*$. The Poisson model for the counts is obtained as the limit when the sizes of the management styles tend to infinity and the proportionality constants in the liquidation intensities tend to zero. Thus, the Poisson model can be seen as a macroscopic approximation for large classes of management styles with rather small monthly liquidation intensities [see e.g. Czado, Delwarde, Denuit (2005) and Gagliardini, Gourieroux (2013) for Poisson approximations in life insurance, and credit risk, applications, respectively]. In the microscopic model the proportionality constants can depend on the fund and capture the fund specific (idiosyncratic) components of the liquidation risk. These idiosyncratic effects are assumed to be eliminated by the aggregation. The aggregation procedure has the advantage of revealing the common factor of interest (frailty). This explains why other HF specific data available in our database, such as the HF returns, are not included in the model [7].

[7] By performing the analysis at the semi-aggregate level of the management style, we also avoid error-in-variable biases. Indeed, while the data on HF lifetimes are rather reliable up to the filtering described in Section 3.1, this is not the case for other individual HF data reported by the fund managers on a voluntary basis. Moreover, during the crisis the hedge funds get the authorization to put their assets, which are difficult to price, in side pockets, and to compute the returns on the remaining part of the portfolio. Due to these side pockets the reported returns can be subject to a significant selectivity bias.

2.2 The common factor dynamics

We complete the model by specifying the conditional distribution of the common factor process. We assume that the frailty variable follows an Autoregressive Gamma (ARG) process. The ARG process is the time discretized Cox, Ingersoll, Ross process [Cox, Ingersoll, Ross (1985)]. The transition of this Markov process corresponds to a noncentral gamma distribution $\gamma(\delta, \eta F_{t-1}, \nu)$, where $\nu > 0$ is a scale parameter, $\delta > 0$ is the degree of freedom of the Gamma transition distribution and parameter $\eta \geq 0$ is such that $\rho = \eta\nu$ is the first-order autocorrelation (see Appendix A.1 for basic results on the ARG process). Since the factor is unobservable, it is always possible to assume $E(F_t) = 1$ for identification purpose. Then, the frailty dynamics can be conveniently parameterized by parameters δ and ρ as stated next.

Assumption A.2: *The conditional distribution of* F_t *given* $\underline{F_{t-1}}$ *and* $\underline{Y_{t-1}}$ *depends on* F_{t-1} *only and is noncentral gamma:*

$$F_t|\underline{F_{t-1}}, \underline{Y_{t-1}} \sim \gamma\left(\delta, \frac{\rho\delta}{1-\rho}F_{t-1}, \frac{1-\rho}{\delta}\right), \tag{2.2}$$

where $\delta > 0$ *and* $0 \leq \rho < 1$.

The independence of the conditional distribution of F_t from the lagged liquidation counts, and the independence of this distribution from the factor values prior to date $t-1$, correspond to the exogeneity and the Markov property, respectively, of the factor process. The ARG specification has several advantages. First, it ensures positive values for the factor, and thus a positive intensity, if $b_k \geq 0$ and $c_{k,l} \geq 0$ for all k, l. Second, the joint model (2.1)-(2.2) for the liquidation counts and the factor is an affine model [see e.g. Duffie, Filipovic, Schachermayer (2003) for continuous time affine processes, and Darolles, Gourieroux, Jasiak (2006) for discrete time]. This allows us to derive analytically the expressions of nonlinear forecasts at any horizon (see Appendix A.2). The single factor assumption is introduced for tractability. It is also in line with empirical findings in the literature. For instance, Carlson, Steinman (2008) regress the aggregate liquidation count for the entire HF market on several time-dependent observable variables related to market conditions and on the lagged aggregate liquidation count, and find only one statistically significant variable. [8] Model (2.1)-(2.2) involves $K(K+2)$ parameters a_k, b_k and $c_{k,l}$ in the liquidation intensities, plus two parameters in the frailty dynamics, that are the degrees of freedom δ and the autocorrelation ρ. The large number of parameters is due to the cross-effects between the management style and frailty, and between the management style and lagged liquidation counts.

3 Hedge funds data on liquidation

3.1 The database and data filtering

The Lipper TASS database [9] consists of monthly returns, Asset Under Management (AUM) and other HF characteristics for individual funds from February 1977 to June 2009. The relevant information for our study concerns the HF status. The database categorizes HF into "Live" and "Graveyard" funds. The "Live" funds are presented as still active. There are several reasons [10] for a fund to be included in the Graveyard database. For instance, these funds *i)* no longer report their performance to TASS, *ii)* are liquidated, *iii)* are merged or restructured, *iv)* are closed to new investors. A HF can be listed in the Graveyard database only after being listed in the Live database. The TASS dataset includes 6097 funds in the "Live" database and 6767 funds in "Graveyard". In our analysis, we consider only

[8] These market conditions are essentially the market index return and the market volatility, the significant observed variable being the S&P 500 return. However, the model in Carlson, Steinman (2008) includes no variable measuring liquidity features, such as measures of counterparty risk.

[9] Tremont Advisory Shareholders Services. Further information about this database is provided on the website http://www.lipperweb.com/products/LipperTASS.aspx.

[10] Graveyard status code: 1=fund liquidated; 2=fund no longer reporting to TASS; 3=TASS unable to contact the manager for updated information; 4=fund closed to new investment; 5=fund has merged into another entity; 7=dormant fund; 9=unknown.

the HF, which are reported as "Live", or "Liquidated" (status code 1) [11]. The latter are 2533 funds. Moreover, in order to account for the time needed to pass from "Live" to "Graveyard" in TASS, we have transferred to the "Graveyard" database the 273 funds of the "Live" database with missing data at least for April, May, June 2009. Among these funds, 23 are considered as liquidated according to this criterion.

We apply a series of filters to the data. First, we select only funds with Net Asset Value (NAV) written in USD. This currency filter avoids double counting, since the same fund can have shares written in USD and Euro for example. After applying the currency code filter, we have 3183 funds in the "Live" base, and 1881 liquidated funds. Second, we select only funds with monthly reporting frequency. Nevertheless, we have also included the funds with quarterly reporting frequency, when the intermediate monthly estimated returns were available. Third, to keep the interpretation in terms of individual funds, we eliminate the funds of funds and, for funds with multiple share classes, we eliminate duplicate share classes from the sample. Finally, we select the nine management styles with a sufficiently large size. These are Long/Short Equity Hedge (LSE), Event Driven (ED), Managed Futures (MF), Equity Market Neutral (EMN), Fixed Income Arbitrage (FI), Global Macro (GM), Emerging Markets (EM), Multi Strategy (MS), and Convertible Arbitrage (CONV). This allows us to use the Poisson approximation for the analysis of the liquidation counts per management style (see Section 2.1). After applying all these filters, we get 2279 funds in the "Live" database and 1520 liquidated funds. The distribution by style of alive and liquidated funds in the database is reported in Table 1.

[Insert Table 1: The database]

The largest management style in the database of alive and liquidated funds is Long/Short Equity Hedge (about 40%), followed by Managed Futures, Multi-Strategy and Event Driven (each about 10%).

[11] Chan, Getmansky, Haas, Lo (2007) have regarded as liquidated all Graveyard funds in status code 1, 2 or 3.

3.2 Summary statistics

Figures 1 and 2 display the variation of the subpopulation sizes and the liquidation rates overtime for different management styles, without distinguishing the age of the HF.

[Insert Figure 1: Subpopulation sizes of HF]

[Insert Figure 2: Liquidation rates of HF]

In Figure 1 we observe the HF market growth between 2000 and 2007, and the sharp decrease due to the 2008 financial crisis. However, the effect of the crisis is less pronounced for HF in the Global Macro style. Figure 2 shows liquidation clustering both with respect to time and among management styles. One liquidation clustering due to the LTCM debacle is observed in Summer 1998 and is especially visible for the Emerging Markets and Global Macro styles. Another liquidation clustering is observed in the 2008 crisis, but did not include the Global Macro style. In fact, for several management styles, the increase of the liquidation rates started before the beginning of the crisis. This finding is likely due to a bubble phenomenon. Indeed, we observe in Figure 1 the increase in the number of funds and then also the number of fund managers. It has likely been accompanied by a decrease of the average skill of the new entering fund managers, which caused the larger observed liquidation rates.

Let us now focus on the age effect. The age of an individual HF is measured since the inception date as reported in TASS. Thus, the age is the official age, that does not take into account the incubation period. We provide in Figure 3 the smoothed nonparametric estimates of the liquidation intensity as a function of age by management style. The estimates are obtained from the Kaplan-Meier estimators of the survival functions [Kaplan, Meier (1958)].

[Insert Figure 3: Smoothed estimates of liquidation intensity]

These estimates have similar patterns, with a maximum at the age of about 4 years. We have computed the estimated liquidation intensities at the ages of 0 and 100 months for each management style. These liquidation intensities are 0.041 and 0.045 for Managed Futures, and 0.045 and 0.084 for Equity Market Neutral, for instance. Thus, the intensity functions of different management styles are not proportional. This suggests that the proportional hazard model should not be used for these HF data.

There can also exist cross-effects of time and age on the liquidation intensity, which are difficult to observe when the hedge fund lifetimes are separately analysed with respect to either time (see Figure 2), or age (see Figure 3). These cross-effects can be detected by means of the Lexis diagrams. Each liquidated fund is marked on the diagram by a dot with the date of death on the x-axis and its age at death on the y-axis. All the funds in the same cohort are represented on the 45^0 line passing through this dot. In particular, the intersection of this line with the x-axis provides the birth date of the funds in this cohort (see Figure 4).

[Insert Figure 4: Lexis diagram]

The Lexis diagrams for three management styles are provided in Figures 5, 6 and 7. In these figures, each star represents a liquidation event in the time-age set of coordinates, and we look for concentration of stars in a band, that is either parallel to the x-axis (age effect), or parallel to the y-axis (time effect), or parallel to the 45^0 line (cohort effect).

[Insert Figure 5: Lexis diagram for Emerging Markets]

[Insert Figure 6: Lexis diagram for Global Macro]

[Insert Figure 7: Lexis diagram for Multi Strategy]

The Emerging Markets strategy represented in Figure 5 features *i)* a concentration of liquidation events around the age of 20 months, *ii)* regularly spaced liquidation events for the cohort born in 1993, and *iii)* another concentration of liquidation events during the crisis of 2008. The Lexis diagram for the

Global Macro strategy (Figure 6) reveals two high time concentrations around 1998 (the LTCM crash) and 2008-2009 (the recent financial crisis), whereas the time concentrations are around January 2003 and the 2008 crisis for the Multi Strategy funds (Figure 7).

Figures 2 and 5-7 show the presence of clustering effects in the liquidation counts across HF management styles. As explained in the Introduction, such clustering effects can be due to either common factors or contagion. In the next section we estimate the multivariate dynamic Poisson model with both frailty and contagion to disentangle the two effects.

4 Estimation of the model

4.1 Model with both frailty and contagion

The likelihood function of the multivariate autoregressive Poisson regression model with shared dynamic frailty is a multi-dimensional integral with a large number of parameters. This makes the numerical optimization of the likelihood function via simulation based methods cumbersome [see e.g. Cappé, Moulines, Rydén (2005) for estimation methods based on Gibbs sampling in nonlinear state space models, and Duffie et al. (2009) for an application in credit risk]. We propose a new informative Generalized Method of Moments (GMM) [Hansen (1982), Hansen, Singleton (1982)] approach for our estimation problem. The moment restrictions are based on the conditional Laplace transform of the liquidation counts. In addition to the intensity parameters, they involve either the parameters in the stationary distribution of the frailty (first-order moment restrictions), or the parameters in the transition distribution of the frailty (second-order moment restrictions). Specifically, the continuum of moment restrictions are (see Appendices B.1-B.2):

$$E\left[\exp\left\{\log(1-v/\gamma_{k,t})Y_{k,t}+v\left(a_k+c_k'Y_{t-1}^*\right)\right\}\right]=\frac{1}{\left(1+vb_k/\delta\right)^{\delta}}, \tag{4.1}$$

for all management styles k, with $k = 1, ..., 9$, and any value of the argument v in a neighbourhood of 0 (first-order moment restrictions), and

$$
\begin{aligned}
& E\left[\exp\left\{\log(1 - v/\gamma_{k,t})Y_{k,t} + \log(1 - \tilde{v}/\gamma_{l,t-1})Y_{l,t-1} + v\left(a_k + c_k' Y_{t-1}^*\right) + \tilde{v}\left(a_l + c_l' Y_{t-2}^*\right)\right\}\right] \\
= \; & \frac{1}{\left[1 + (v b_k + \tilde{v} b_l)/\delta + (1-\rho) v \tilde{v} b_k b_l / \delta^2\right]^{\delta}}, \qquad (4.2)
\end{aligned}
$$

for all pairs (k, l) and any values of the arguments v, $\tilde{v}$ in a neighbourhood of 0 (second-order moment restrictions). The implementation of the corresponding GMM estimator is discussed in Appendix B.3.

We estimate the model on the sample from January 1996 to June 2009 in order to ensure that the size of each of the nine management styles is sufficiently large, specifically at least 50 funds. In Table 2 we display the estimated intercept parameters a_k and factor sensitivities b_k along with their standard errors. The estimated contagion matrix is provided in Table 3, where we display only the statistically significant contagion coefficients at the 10% level. The estimated parameters of the frailty dynamics are $\hat{\delta} = 0.59$ (with standard error 0.34) and $\hat{\rho} = 0.74$ (with standard error 0.20). The standard errors of the GMM estimates are computed using the asymptotic distribution.

[Insert Table 2: Estimated intercepts and factor sensitivities in the model with contagion and frailty]

[Insert Table 3: Estimated contagion parameters in the model with contagion and frailty]

The estimated factor sensitivities are all statistically significant. Only 11 (resp. 13) estimated contagion parameters in Table 3 are statistically significant at the 5% (resp. 10%) level. The contagion scheme for the model with contagion and frailty is displayed in Figure 8, where an arrow from style l to style k corresponds to an estimate of parameter $c_{k,l}$ that is statistically significant at the 5% level.

[Insert Figure 8: The contagion scheme for the model with contagion and frailty]

In Figure 8 we observe that contagion occurs along specific directions, such as Multi Strategy $\rightarrow$

Equity Market Neutral $\to$ Event Driven $\to$ Fixed Income Arbitrage $\to$ Emerging Markets, without any evidence of contagion in the reverse direction. The Long-Short Equity Hedge management style is the largest in our dataset in terms of both number of funds (see Table 1) and total AUM (about 40% of our total database AUM). The lack of a central role of Long-Short Equity Hedge funds in the contagion scheme in Figure 8 confirms the idea that systemic relevance is not necessarily associated with size.

In Tables 2 and 3, the estimates of the intercepts a_k and the rows c_k' of the contagion matrix differ significantly across management styles k. When another model with the fund age and management style as the explanatory variables is fitted to the data (see Figure 3), the age variable partly captures the effect of the time-varying common factor and lagged liquidation counts, which appeared as the explanatory variables in model (2.1), with different impacts to the various management styles. Thus, the results in Tables 2 and 3 are compatible with the findings in Figure 3, and support the evidence that a proportional hazard specification without any time-varying explanatory variables is not adequate.

4.2 Model with contagion only

In order to assess the importance of introducing the common factor to explain liquidation clustering, in this section we estimate a model with pure contagion. The specification is:

$$Y_{k,t} \sim \mathcal{P}\left[\gamma_{k,t}(a_k + c_k' Y_{t-1}^*)\right], \quad k = 1, ..., 9, \tag{4.3}$$

and corresponds to model (2.1) with $b_k = 0$ for any management style k. It is a multivariate Poisson regression model, with the observed lagged adjusted liquidation counts as explanatory variables. The lagged counts capture the liquidation clustering effects and their diffusion between and within management styles. The model involves 9 intercept parameters a_k, with $k = 1, ..., 9$, and a matrix of 81 contagion parameters $c_{k,l}$, with $k, l = 1, ..., 9$. The parameters of the Poisson regression model are estimated by the Maximum Likelihood (ML) [Cameron, Trivedi (1998)]. The estimated values of the intercepts are given in Table 4 with standard errors in parentheses. The estimated contagion matrix is provided in Table 5, where we display only the statistically significant contagion coefficients at the 5% level.

[Insert Table 4: Estimated intercepts in the pure contagion model]

[Insert Table 5: Estimated contagion parameters in the pure contagion model]

The contagion matrix is represented as a network in Figure 9, where any estimated contagion coefficient in Table 5, that is statistically significant at 5% level, is represented by an arrow.

[Insert Figure 9: The contagion scheme for the pure contagion model]

All strategies seem now interconnected either directly, or indirectly through some multistep contagion channels. A contagion scheme featuring this property would correspond to a complete structure in Allen, Gale (2000) terminology. The structure of the estimated contagion matrix provides interesting information on the possible model misspecification when the frailty is omitted. We observe the special roles of the Fixed Income Arbitrage and Long/Short Equity Hedge styles, which influences directly, respectively is influenced by, most of the other styles. However, some estimated contagion parameters likely indicate a misspecification of the model without frailty and lead possibly to misleading interpretations. For instance, we get a large value 0.64 of the contagion parameter from Fixed Income Arbitrage to Long/Short Equity Hedge. Such a causal effect is unlikely as the Fixed Income Arbitrage strategies invest in bonds and, when the associated managers deleverage their portfolios, the impact on Long/Short Equity strategies invested in stocks is expected to be small. In the supplementary material available online we consider some test statistics based on the model residuals that can be used as diagnostic tools for the hypothesis of no frailty. The results of the tests show that the Poisson model with pure contagion is not able to fully reproduce the observed clustering in the liquidation counts. This finding supports the hypothesis of the misspecification of the contagion model without frailty in favor of the complete specification with both contagion and frailty estimated in Section 4.1. We use the estimates of such complete specification for the empirical analysis in the rest of the paper.

4.3 The relative importance of frailty and contagion

The relative effect of contagion and frailty on liquidation risk can be measured by using the variance decomposition given in the next proposition (see Appendix A.4 for the proof).

Proposition 1: *The variance-covariance matrix of the liquidation count vector Y_t can be decomposed as:*

$$\begin{aligned} V(Y_t) &= diag[E(Y_t)] + CV(Y_t)C' + (1/\delta)bb' \\ &\quad +(1/\delta)\rho C(Id-\rho C)^{-1}bb' + (1/\delta)\rho bb'(Id-\rho C')^{-1}C', \end{aligned}$$

where $diag[E(Y_t)]$ denotes the diagonal matrix with diagonal elements corresponding to the elements of the vector of expected liquidation counts $E[Y_t] = (Id - C)^{-1}(a + b)$, and a and b denote vectors with elements a_k and b_k, respectively.

The decomposition formula in Proposition 1 is valid under the stationarity conditions on the contagion matrix C and the autocorrelation ρ of the frailty. These conditions are as follows (see Appendix A.3): the eigenvalues of the contagion matrix C are less than 1 in modulus, and the autocorrelation ρ of the frailty is less than 1. In the decomposition of the historical variance-covariance matrix of the liquidation counts, the first term $diag[E(Y_t)]$ in the right hand side corresponds to the variance in a Poisson model with cross-sectional independence. The sum of the first and second terms provides the expression of the variance in a model including contagion, but without frailty. The third term $(1/\delta)bb'$

captures the direct effect of the exogenous frailty. The remaining terms accommodate its indirect effects through contagion, namely, the amplification of the frailty effect due to the network. This variance decomposition is written in an implicit form as the system in Proposition 1 has to be solved to get the expression of $V(Y_t)$ as a function of the model parameters.

Let us now assess the magnitude of the terms in the variance decomposition by using the estimated model in Section 4.1. To do that we consider a portfolio of Liquidation Swaps (LS) written on the individual hedge funds, which is diversified with respect to the management styles. The liquidation swap for management style k pays 1 USD for each fund of style k that is liquidated in month t. The payoff of the LS portfolio at month t is $e'Y_t$, where $e = (1, 1, ..., 1)'$ is a $(9, 1)$ vector of ones. To ensure the time-invariant diversification, the portfolio of LS has to be appropriately rebalanced when a liquidation occurs. By using the variance decomposition, we can evaluate the percentage of portfolio variance $e'V(Y_t)e$, due to the underlying idiosyncratic (i.e., management style specific) Poisson shocks, contagion and frailty, respectively. The decomposition is displayed in Table 6.

[Insert Table 6: Decomposition of the variance]

The largest contribution to the portfolio variance comes from the frailty process, either through a direct effect (64.30%), or through an indirect effect via the contagion network (24.06%). The remaining part of portfolio variance is explained by the underlying Poisson shocks (6.54%) and the direct contagion effects (5.10%). Even though the direct effect of contagion is modest, the network plays an important role in amplifying the effect of the exogenous frailty.

5 Liquidity risks and HF liquidation dependence

In this section we propose an interpretation of the estimated model for HF liquidation counts in terms of funding and market liquidity risks. Specifically, in Section 5.1 we relate the common factor sensitivities of the different management styles with their exposures to funding liquidity risk. Section 5.2 discusses the estimated contagion scheme. Finally, Section 5.3 considers the filtering of the unobservable common factor and its relation with traditional observable measures of funding liquidity risks.

5.1 Sensitivities to the systematic risk factor

As discussed in the Introduction, the unobservable dynamic frailty is likely a measure of economy wide funding liquidity risk. This interpretation is supported by a careful analysis of the estimated sensitivities in Table 2 across management styles. For a given management style, the liquidity features are twofold: the portfolio can be invested in more or less liquid assets (i.e. assets without a large haircut in case of fire sales), and the strategy can require a longer or shorter horizon to be applied. In this respect, Global Macro and Managed Futures portfolios are invested in liquid assets; they can offer to investors weekly, or even daily liquidity conditions, and have small sensitivity coefficients 0.33 and 0.63, respectively. At the opposite, the Event Driven strategies are essentially looking for positive outcomes in mergers and acquisitions, which can only be expected in a medium horizon. They have less interesting, generally quarterly, liquidity conditions and the factor sensitivity coefficient 1.39 is the second highest one. Similar remarks can be done for other management styles.

The discussion above is completed by comparing the factor sensitivities to the average redemption frequencies and leverage in each management style, which are displayed in Table 7.

[Insert Table 7: Redemption frequency, leverage and factor sensitivity]

While the majority of funds across management styles allow for redemptions on a monthly basis or more often, we observe some differences, especially in the Event Driven management style, where the redemption frequency is often close to 3 months. It has been observed that "hedge funds with favorable redemption terms differ significantly in terms of their appetites for liquidity risks" [see Teo (2011)]. In Table 7 we observe a significant negative link between the factor sensitivity and the proportion of hedge funds with a redemption frequency of one month or less (resp. of HF reporting the use of leverage). This link is likely explained by the type of assets introduced in the HF portfolio. For instance, as already remarked the funds in the management styles Managed Futures and Global Macro are invested in very liquid assets. Thus, they can easily propose good redemption frequencies and use leverage without being too sensitive to the common factor. However, to attract investors, some managers in other strategies may propose "favorable" redemption conditions and simultaneously post high returns obtained by taking an excessive liquidity risk. This is likely the case for some HF in the Long Short Equity category, where the favorable announced redemption and the usual high leverage are not in line with the very high exposure to the funding liquidity risk factor. [12] The link between the frailty sensitivities and the redemption frequencies is confirmed by the correlation between the former and the proportion of favorable redemption conditions (less than one month) equal to -0.27, passing to -0.56 when the Long Short Equity category is not considered. Similarly, the correlation between the frailty sensitivities and the proportions of funds reporting the use of leverage is -0.32. To summarize, the sensitivity coefficients measure the funding liquidity risk exposures of the different management styles, and these exposures are related to the management of gates and leverage. This interpretation will be further discussed in Section 5.3 when filtering the factor.

5.2 The contagion scheme

Let us now discuss carefully the scheme in Figure 8. This estimated scheme shows a classification of management styles into four categories: *i)* Funds mainly invested in fixed income products and us-

[12] In this respect, 43% of the Long Short Equity Hedge funds managers report zero leverage (Table 7), which is not compatible with their announced management style. There does not exist reliable data on leverage of hedge funds. In general they are static and not detailed enough to distinguish the short and long positions of the HF. An exception is Ang, Gorovyy, van Inwegen (2011), who get such a database for a subpopulation of HF from a fund of hedge funds. However, even if this subpopulation is almost representative of the TASS population, the definition of the management styles differ.

ing high leverage, that are Fixed Income Arbitrage, Managed Futures, Emerging Markets and Global Macro. *ii)* Funds mainly invested in equities, such as Equity Market Neutral, Long/Short Equity Hedge and Event Driven. *iii)* Funds in the Convertible Arbitrage management style, in which the convertible products have features of the corporate bonds and associated stocks. *iv)* Funds in the Multi-Strategy management style, with portfolios including subprimes and equities. This classification clearly differs from other classifications in the literature such as the one introduced by Morningstar, in which the funds are divided into Directional Traders, Relative Value, Security Selection and Multiprocess, respectively [see MSCI (2006) and Agarwal, Daniel, Naik (2009), Appendix B]. That classification is based on the type of portfolio management strategy followed by the fund. For instance, Security Selection managers take long and short positions in undervalued and overvalued securities, while trying to reduce the systematic market risk. However, they can invest in more or less liquid assets, and introduce different levels of leverage. By comparison, our classification is clearly funding liquidity risk oriented and revealed by the HF liquidation data.

The causal scheme in Figure 8 has been estimated from a sample that contains several funding liquidity crises. Depending on the specific crisis, the exogenous shocks to funding liquidity may impact some management styles more than the others. Then, if the shock hits for instance the Fixed Income Arbitrage style, that shock will have a direct effect on the Emerging Markets, but not on the styles that are before the Emerging Markets in the causal chain, such as the Event Driven and the Equity Market Neutral styles. As an illustration, let us discuss the causal scheme in relation with the recent subprime crisis and the associated lack of market liquidity in various classes of assets. In 2007, there has been an increase of expected default rates for mortgages, followed by an increase of margin calls for credit derivatives. The Multi-Strategy funds, that invest in subprimes and in liquid market neutral strategies, needed cash in order to satisfy the margin requirements. It was then natural for them to liquidate the most liquid part of their portfolios, i.e. the equity strategies. This massive deleveraging had a direct effect on the stock prices, increasing the market liquidity risk. At the beginning this effect was not observed in the stock indices, but mainly in the relative performances of individual stocks: the high-ranked stocks becoming low ranked and vice-versa, since the strategies followed by Equity Market Neutral and Long/Short Equity Hedge funds, for instance, are less sensitive to the market [see Patton (2009) for a careful analysis]. This dislocation effect of stock prices has impacted all the equity strategies, including the Event Driven funds. The associated M&A strategies have transformed the short-term shocks into long-term shocks. This explains the key (systemic) role of the Event Driven management style, which creates the link between the shocks to stock markets and the shocks to fixed income markets.

5.3 The systematic factor

Three types of approaches have been followed in the literature to measure and analyze the funding liquidity risk and its evolution (from return data):

i) The first approach considers directly the refinancing costs. The Treasury-Eurodollar (TED) spread, equal to the difference between the 3-month Eurodollar LIBOR rate and the 3-month Treasury bill rate, is such a measure of refinancing cost frequently considered in the literature [see e.g. Gupta, Subrahmaniam (2000), Boyson, Stahel, Stulz (2010), Teo (2011)].

ii) Alternatively, one could consider a direct measure of market liquidity, such as a bid-ask spread, and use the link between funding and market liquidity emphasized in Brunnermeier, Pedersen (2009) [see e.g. Goyenko, Subrahmaniam, Ukhov (2011)].

iii) Finally, liquidity is analyzed from its effects on asset prices. In this respect, Vayanos (2004) suggests to measure the liquidity premium between two assets with similar characteristics, but of different liquidities. These assets can be for instance thirty-year Treasury bonds just issued (on-the-run), or issued three months ago. They have the same cash flows, but the on-the-run bonds are much more liquid [see e.g. Fontaine, Garcia (2012)]. Another example of such assets are the German bonds (i.e. the bunds) and those issued by the Kreditanstalt fur Wiederaufbau (KfW), a German agency whose bonds are explicitly guaranteed by the Federal Republic of Germany [see e.g. Gourieroux, Monfort, Pegoraro, Renne (2014)].

In our analysis of HF survival, we follow an alternative approach. Since the hedge funds are often invested in derivatives and use a high leverage, they can be very sensitive to the funding liquidity risks. Thus, we expect the common factor in the analysis of hedge fund survival to be a proxy for funding liquidity risk. This section further motivates the funding liquidity risk interpretation of the frailty developed in Subsection 5.1. We first filter the unobservable factor path. Then, we investigate how this factor is related with other funding liquidity proxies introduced in the literature.

i) Filtering of the factor

The Poisson model with both frailty and contagion is a nonlinear state space model, which requires adequate methods to filter the unobservable factor. Since the joint process $(Y_t', F_t)'$ of observable and unobservable variables is affine (see Appendix A.2), the Bates filter [Bates (2006)] might be used to update the conditional Laplace (or Fourier) transform of the filtering distribution. However, the implementation of the Bates filter requires, at each iteration, the evaluation of a numerical integral of dimension equal to the number of observable variables. In our model, the vector of observations (the liquidation counts) is of dimension nine, which makes the implementation of the Bates filter numerically unfeasible. In the online supplementary material we build on the insight of Bates (2006) and propose a new filter for our model, which takes advantage of the affine property of the ARG frailty dynamics, but requires only a few one-dimensional numerical integrals to be computed at each iteration. The filter is based on the idea of approximating any conditional distribution of the frailty given the available information by means of a distribution in the gamma family (see the online supplementary material for a description of, and a comparison with, alternative filtering approaches). The filtered path of the frailty is displayed in Figure 10.

[Insert Figure 10: Filtered path of the frailty]

The frailty features a rather stable path between 1996 and 2006, with spikes at the end of 2001 (the 9/11 terrorist attack), the end of 2002 (the market confidence crisis due to the internet bubble), ... The frailty path features an upward trend over the years 2007 and 2008 (the recent financial crisis), and decreases rapidly afterwards.

ii) The factor interpretation

The TED spread and the VIX are commonly used as measures of capital availability in the economy [see e.g. Goyenko (2012)]. As already mentioned, the TED spread is a measure of the refinancing cost on the clearing houses. It is introduced to capture a part of the rollover funding liquidity risk. The VIX is a weighted average of the implied volatility in the S&P index options. This index measures the aggregate volatility of the stock market as well as the price of this volatility. In our framework, it is introduced to capture the magnitude and the cost of leverage. Ang, Gorovyy, van Inwegen (2011) find that the dynamics of the HF net leverage (i.e., the difference between short and long positions) and gross leverage (i.e., the sum of these positions) are explained by the VIX and the TED. Adrian, Shin (2010) report similar findings for other financial intermediaries. The VIX is also introduced to capture the cost of leverage, i.e., the margin funding liquidity risk. Indeed, for listed derivatives, but also for OTC derivatives, the margin calls depend on the volatility of the underlying index (and are independent of the riskiness of the fund). Thus, an increase of the volatility, proxied by the VIX, implies an increase of the margin calls and an increase of the cost of leverage. Moreover, in crisis periods HF managers turn to VIX futures for volatility hedging and their short positions on VIX futures were for instance so extreme on February 27, 2013, that the market was close to a significant squeeze. The margin calls on VIX futures are directly written on the VIX itself. The time series of the TED spread and the VIX are displayed in Figure 11. The data are obtained from the Federal Reserve Board's website for the TED spread, and from the Chicago Board Options Exchange (CBOE) website for the VIX.

[Insert Figure 11: Observable liquidity indicators]

We estimate the regression:

$$\begin{aligned}\hat{F}_t &= I(VIX_t \leq c)\,(\beta_1 + \beta_2 TED_t + \beta_3 TEDL_t + \beta_4 VIX_t + \beta_5 VIXL_t + \beta_6 SPR_t) \\ &\quad + I(VIX_t > c)\,(\gamma_1 + \gamma_2 TED_t + \gamma_3 TEDL_t + \gamma_4 VIX_t + \gamma_5 VIXL_t + \gamma_6 SPR_t) + e_t, \end{aligned} \tag{5.1}$$

where the explained variable $\hat{F}_t$ is the filtered value of the frailty. In addition to the current values of the TED spread and the VIX, the regression includes lagged observations via the average value of the TED spread in the previous quarter (TEDL) and the average value of the VIX in the previous 12 months (VIXL), to potentially capture the impacts of the associated innovations. The regression also includes the spread SPR between the BAA and AAA yields from the FRED database at the Federal Reserve Bank of St. Louis (see Figure 11, lower panel). It is often stated that part of this spread is unrelated to credit risk, and it is due to the lower liquidity of the corporate bonds in the more risky rating classes. While the empirical literature focuses mostly on linear relationships between the unobservable factor and the observed explanatory variables, we expect to detect nonlinear effects. Indeed, it is widely believed that hedge funds provide liquidity to the markets, especially to markets of assets with high degree of information asymmetry [see e.g. Agarwal, Fung, Loon, Naik (2007), Brophy, Ouimet, Sialm (2009)]. However, this occurs in the standard situation of reasonable funding liquidity costs. In this "good equilibrium", the hedge funds provide liquidity and are invested in rather illiquid assets with high leverage. But, as noted in Ben-David, Franzoni, Moussawi (2012), when the refinancing costs increase, hedge funds reallocate their portfolios, reduce their equity holdings and try to diminish their leverage in order to anticipate the consequences of possible outflows. In this "bad equilibrium", the hedge funds are liquidity seekers. This double equilibrium is captured by the threshold on the VIX, with the estimated monthly percentage value $c = 25$. We use a switching regime regression by allowing for different coefficient values in the low and high volatility regimes. The estimates of the coefficients in regression (5.1), as well as in restricted specifications, are displayed in Table 8.

[Insert Table 8: Regression of the frailty on observable variables]

The estimated regressions show that a large fraction of the common factor (i.e. about 70%) is explained by the proxies for funding liquidity risk introduced in the specifications. The coefficient of TED is statistically significant (at the 1% level) and larger in the "bad equilibrium", while in the "good equilibrium" the lagged value TEDL has a larger effect. The effect of the volatility index passes through the lagged value VIXL, which has a negative and statistically significant coefficient. This negative effect of VIXL is more pronounced in the "bad equilibrium". The negative regression coefficient for the lagged VIX has to be interpreted in relation with the interaction between VIX and TED. [13] Indeed, when we regress the frailty on a constant, VIX and VIXL, we find a positive coefficient on the current VIX, and a negative coefficient of about the same magnitude on the lagged VIX. Thus, the short term overreaction to a shock to the VIX disappears almost completely in the long term. In other words, the frailty reacts to the innovations in the VIX. When the TED and its lagged values are included in the regression, the short term effect of VIX is captured by the TED, and the associated regression coefficient becomes statistically non-significant. Finally, we observe that the marginal effect of the spread SPR is smaller in the high volatility regime.

Even if observable proxies for the funding liquidity risk, such as the TED spread and the VIX, can explain a substantial part of the frailty effect, Table 8 shows that the frailty is not entirely captured by these observable proxies. This finding highlights the usefulness of including an unobservable common factor in the liquidation intensity specification. By identifying, a priori, the systematic risk factor

[13] The correlation between VIX and TED is 0.33 in the low volatility regime, and 0.57 in the high volatility regime.

with some observable variables, we would implicitly disregard the risk associated with the difference between the frailty and the observable proxies.

6 Stress-tests

The estimated model with dynamic frailty and contagion can be used for portfolio management of a fund of funds, and computation of reserves, etc. In this section, we illustrate how to implement the prediction of future liquidation counts and the stress-tests for liquidation risk. We consider a portfolio of HF with fixed style sizes and compare the distribution of the future liquidation counts in the unstressed and stressed situations. The future counts are subject to a double uncertainty, that is the drawing of the idiosyncratic risks in the Poisson conditional distribution, and the stochastic evolution of the exogenous dynamic frailty. The analysis in the unstressed situation corresponds to the prediction of any (nonlinear) function of the liquidation counts and serves as a benchmark. In the conditioning set, the unobservable current value F_t of the frailty is replaced by the filtered value [see Section 5.3 i)]. The stress can be designed in the following ways:

i) We can stress the current factor value by setting $F_t = q_\alpha$ in the conditioning set, where q_α is the quantile of the estimated stationary distribution of the frailty F_t at level α. By choosing $\alpha = 95\%$, or 99%, we consider an extreme scenario with a large transitory shock to the underlying funding liquidity risk factor at month t.

ii) Alternatively, we can change parameters values, by either "increasing" the matrix of contagion C, or by increasing the value of the frailty persistence parameter ρ. This stress scenario will increase the liquidation risks by amplifying the impact of the exogenous shock by contagion, and by introducing serial clustering in the exogenous shocks, respectively.

At month t, the stress scenario is characterized by the observed liquidation counts and the possibly stressed values of the factor in month t and the parameter values. Next, the future paths of both the factor and the liquidation counts could be simulated conditional on this information and used to compute the conditional distribution of the liquidation counts at any horizon of interest. In particular, we focus on the term structures of the expected liquidation counts, and of the volatility and overdispersion of these counts. Actually, these term structures can be derived in closed form in our model by using the exponential affine property of the joint process of frailty and liquidation counts (see the supplementary material). Our stress test analysis is dynamic and fully accounts for both liquidation counts and the exogenous frailty dynamics. Therefore, it sharply differs from the stress test analysis in models with time-varying observable variables, in which a crystallized scenario for the future factor path is assumed. Such a stress test would disregard the liquidation risk dependence induced by the exogenous factors and would generally undervaluate the future risk.

We consider three sets of stress scenarios:

S.1: The current factor value F_t is increased from the filtered value to the 95% quantile of the historical distribution. The parameter values correspond to the estimates of Section 4.1. By conditioning on an extreme event on the current value of the systematic risk factor, the expected liquidation counts are in line with the measures for systemic risk such as the CoVaR [Adrian, Brunnermeier (2011)], or the marginal expected shortfall [Acharya et al. (2010)]. The main difference is in the definition of the conditioning set including the unobservable factor and the observable liquidation counts, instead of including the observed return of a market portfolio.

S.2: The contagion matrix is changed from $\hat{C}$ to $2\hat{C}$, where $\hat{C}$ is the estimate of Section 4.1. The other parameter values are kept constant and equal to the estimates of Section 4.1. The current factor value F_t is set equal to the filtered value.

S.3: The frailty autocorrelation is increased from $\rho = 0.74$ (corresponding to the estimate in Section 4.1) to $\rho = 0.90$. The other parameter values are kept constant and equal to the estimates of Section 4.1. The current factor value F_t is set equal to the filtered value.

For all stress scenarios, the filtered value of the frailty and the vector of observed liquidation counts in the conditioning set correspond to the last month of the sample, i.e. June 2009. The category sizes correspond to this date as well. In Figures 12, 13 and 14 we display for the nine management styles the impact of stress scenarios S.1, S.2 and S.3, respectively, on the term structures of the conditional expectations of liquidation counts. [14]

[Insert Figure 12: Term structure of expected liquidation counts when stressing the current factor value]

[Insert Figure 13: Term structure of expected liquidation counts when stressing the contagion matrix]

[Insert Figure 14: Term structure of expected liquidation counts when stressing the frailty persistence]

In each figure, the squares represent the term structures of the expected liquidation counts before stress, that are the same in each scenario. As the horizon increases, the term structure converges to the unconditional expectation of the liquidation count, for each management style. The unstressed term structures are upward sloping since the current month, i.e. June 2009, corresponds to a period with few liquidation events in any management style and a small frailty value compared to the historical average. The circles represent the term structures of the expected liquidation counts after the shock. The three types of shocks have very different effects on the term structures. The shock to the current factor value in stress scenario S.1 is a transitory funding liquidity shock, with different impacts in the short run with respect to the management style (see Figure 12). Its effect decays rather quickly and disappears after about 12 months. The results of these stress tests are compatible with the liquidity interpretation of the unobservable factor. We observe an immediate effect of the shock on the highly exposed strategies of the Long/Short Equity Hedge management style, whereas the effect is clearly lagged, and mainly due to contagion, for the Fixed Income Arbitrage strategies featuring small factor betas (see Table 2). The magnitude of the impact on the term structure depends on the conditioning

[14]Figures with the term structures of conditional volatility and overdispersion of liquidation counts are provided in the supplementary material.

information, i.e. on the liquidation counts at the month of the stress. In stress scenario S.2, the change in the contagion matrix is a permanent shock. In Figure 13, there is no important effect in the short run, but the long run behaviors of the models with and without the shock in the contagion matrix significantly differ for all styles, except for Global Macro. Indeed, the elements in the row of the estimated contagion matrix (Table 3) for that management style are zero. We conclude that there is no significant contagion effect impacting the Global Macro style. Therefore, the stress in scenario S.2 is irrelevant for the distribution of liquidation counts in that management style. Finally, when the frailty persistence parameter is shocked in stress scenario S.3, we observe an increase in the time at which the long run expected values of the liquidation counts are attained (Figure 14).

A large part of the literature on financial contagion applied to asset returns defines contagion as revealed by an increased correlation of asset returns during crisis periods [see e.g. King, Wadhwani (1990)]. Then, the idea is to test for significant changes in some parameters between the crisis and non-crisis periods [see e.g. Forbes, Rigobon (2002)]. The above stress-tests analysis shows that there exist different ways to obtain extreme liquidation counts in some months. This can be due to an extreme exogenous shock possibly amplified by contagion. It can also be due to a standard exogenous shock and a significant change in the contagion matrix. These two situations are clearly different. In the first case, we get an "exogenous crisis" and this exogenous crisis can arise without modifying the contagion matrix C, that is, the speed of propagation of the shocks. In the second case, there is a change in the structure and speed of contagion, which is more in line with the above mentioned financial literature. Such a change can be due to either a change in the behaviour of the fund managers, or of the supervisor for systemic risk. [15] To summarize, the model considered in this paper provides a framework to highlight that the policy maker and the supervisor may have to distinguish between exogenous crises, due to exogenous shocks, and endogenous crises, due to changes in the contagion

[15] Time variation in the contagion matrix could be made endogenous in the model. Such an extension is beyond the scope of this paper.

matrix. To diminish the probability of an exogenous crisis, the policy maker and the supervisor have to control the extreme exogenous risks, that is, the distribution of factor F_t. As an illustration let us recall the example of the Asian flu. In order to control the exogenous risk, the authorities in charge of health supervision will reduce the population of birds. In order to diminish the probability of endogenous crisis, the authorities would try to limit the contacts between humans. The usual practice of stress tests is to consider only the exogenous shocks with a given contagion scheme, and to define as carefully as possible the sources of the shocks. For the analysis of systemic risk, it is more relevant to shock directly the underlying common risk factor instead of introducing shocks to the idiosyncratic risks of each management style hidden in the Poisson (conditional) distribution. By selecting the factor we introduce a common shock to all management styles with different weights given by the frailty beta coefficients. As the policy maker and the supervisor do not have models with frailty and contagion at hand, they have never implemented stress testing to the contagion scheme itself. Nevertheless, during the crisis, some regulators have imposed a change of the contagion scheme, for instance by allowing to isolate a part of the hedge funds portfolios invested in illiquid assets into the so-called side pockets.

Let us now analyse the dynamics of the stress effects over a given horizon. We focus on the stress to the current factor value similarly as in scenario S.1, and at the one-month horizon.

[Insert Figure 15: Time series of prediction and stress tests for liquidation rates]

In Figure 15 we display four time series for each management style: *i)* The time series of the conditional expected liquidation rate over the period from January 1996 to June 2009. In each month, the conditioning information corresponds to the filtered value $\hat{F}_t$ of the frailty and the observed liquidation counts Y_t. *ii)* The time series of the conditional 95 percentile of the liquidation rate, with the same conditioning information as in the previous series. *iii)* The time series of the conditional 95 percentile of the liquidation rate, with the conditioning information corresponding to the observed liquidation counts and a stressed value of the frailty. This stressed value is the 95 percentile of the conditional distribution of the frailty given the current filtered value $\hat{F}_t$. *iv)* The time series of the realized liquidation rate. We focus on the liquidation rate, instead of the liquidation counts, to eliminate the effect of the time-varying sizes of the management styles. Moreover, the liquidation rate parallels the portfolio default probability usually considered in credit risk analysis. [16] The series of the conditional expectation and conditional quantile in *i)* and *ii)* provide the predictions of the mean and an extreme value, respectively, of the one-month-ahead liquidation rate. In particular, the conditional quantile corresponds to the Value-at-Risk (VaR) measure, and serves as a basis to compute the reserves. The difference between the conditional quantile and the conditional expectation is the analogue of the economic capital. The use of the current filtered value of the frailty in the conditioning information corresponds to an unstressed situation. To get a stressed value of the VaR, we increase the frailty value in the conditioning set from the filtered value to the 95 percentile of the conditional distribution given this filtered value. The stressed factor value is defined with respect to the conditional distribution, and not the historical distribution, in order to ensure that the stressed factor value is an extreme scenario compared to the current environment also during a crisis period. The difference between the stressed and unstressed conditional quantiles corresponds to an additional reserve, that is, a second capital buffer.

In Figure 15 the series of conditional quantiles vary over time and increase substantially during the recent financial crisis for most management styles, pointing to the higher liquidation risk in this period. The panels show that the increase of reserves (VaR and stressed VaR) based on our model starts gradually from the end of the first quarter of 2007, that is, several months before the freezing announcements of some funds by Bnp Paribas and AXA. The dates at which this increase starts differ by one or two months across the management styles, due to the contagion effect. This rather early increase of the need for reserves is due to the introduction of the common frailty in the model. This frailty captures the preliminary effect of the crisis once a first management style is impacted, and is

[16]The portfolio loss is proportional to the portfolio default frequency, if the portfolio has uniform exposures and the recovery rate, assumed independent of the default event, can be replaced by its expectation, that is, one minus the expected loss given default.

then transmitted to the other management styles. A model without dynamic frailty would increase the reserves once a significant number of styles have been impacted, that would be too late. Thus, our approach is in line with the recent emphasis on the regulation of systemic factors, especially the smoothing of the reserve increase at the top of the cycle in order to diminish the procyclical effect of the regulation on the required capital [Basel Committee on Banking Supervision (2010)]. In Figure 15, we also observe that the unstressed conditional quantile admits a coverage close to the nominal one, with occurrences of a realized liquidation rate above the conditional quantile in line with the 5% nominal probability. The increase in the conditional quantile due to the stress to the current factor value is rather constant in time. For instance, it is equal to about 0.02 for the Convertible Arbitrage hedge funds. The realized liquidation rate never exceeds the stressed conditional quantile in our sample. The series of unstressed and stressed predictions for the Global Macro hedge funds are smoother than for other management styles, since the Global Macro style features a small sensitivity to the dynamic frailty and is not impacted by significant contagion effects (see Tables 2 and 3).

7 Concluding remarks

In this paper we develop a new methodology to analyse the dynamics of liquidation risk dependence in the hedge fund industry. The autoregressive Poisson model with dynamic frailty is especially advantageous as it allows for distinguishing the effect of the exogenous shocks, which affect directly the liability component of the balance sheet, from the endogenous contagion effects, which pass through the asset component. The common factor, the sensitivities to this factor, and the contagion scheme can all be interpreted in terms of the liquidity risks. The underlying factor is related nonlinearly to the standard proxies of rollover and margin funding liquidity risks with two endogenous regimes. In the first regime, when the aggregate funding liquidity is not tight, the hedge funds are liquidity providers, while in the second regime, when the aggregate funding liquidity is tight, hedge funds become liquidity seekers. The sensitivities to the factor represent the funding liquidity risk exposures of the different management styles and are linked with the redemption frequencies and gate management by the fund managers. Finally, the causal scheme captures a part of the spiral effect highlighted in Brunnermeier, Pedersen (2009), in which market liquidity and funding liquidity are mutually reinforcing, with a shock to one management style propagating into the others. Only a specification with both frailty and contagion allows for performing relevant stress tests. The recent regulation has to define the reserves to cover extreme systematic risks. To do that the regulator has to stress the systematic factor without stressing the hedge fund specific risk factors, and to account properly for contagion phenomena. That analysis leads to the term structure of the reserves for different horizons and the identification of the systematically important management styles.

The frailty and contagion components could also be disentangled in a more general parametric dynamic framework. For instance, the model can include more than one factor, or the lagged effects of these factors on the liquidation intensity. In the latter case, the effect of the (multiple) frailty will also be observed at some lags. Moreover, the model can account for an autoregressive order larger than one in the past liquidation counts. The relative magnitude of the frailty and contagion components will depend on the selected number of factors and lags.

It is known that systemic risk can be due to a significant shock to a common factor amplified by the contagion. Models with both frailty and contagion can reveal the main channel of systemic risk, namely the exogenous shock and/or contagion, and can help develop accurate strategies against systemic crises. For the hedge fund sector, such an analysis would have to include jointly the market risk, that is the returns, and the liquidation risk, that is the lifetime variables. In this respect, our work completes the systemic risk analysis based on returns developed in a series of recent papers including Sadka (2010), Boyson, Stahel, Stulz (2010), Akay, Seniuz, Yoldas (2011), Brown et al. (2011), Billio et al. (2012), Bali, Brown, Caglayan (2012), and the analysis of comovements between the cash inflows and outflows [Sialm, Sun and Zheng (2012)], especially since the common factor for HF liquidation differs from the factors usually exhibited for HF returns. In such analysis, it would be interesting to study the causality between liquidation (or distress) events and returns. Indeed, in August 7, 2007, BNP Paribas has frozen three investment funds, suspending the calculation of their net asset values. The news of this event is often considered as an initial step in the recent financial crisis, including the market freeze for liquidity and the failure of Bear Stearn's or Lehman Brother's. However, these funds have been defrozen later on, and not bankrupted. Thus, some distress events can be advanced indicators of losses of values on other assets. Of course, such joint analysis might be performed for an enlarged perimeter, including hedge funds, stocks, corporate bonds, but for the reasons mentioned before clearly distinguishing the distress events (liquidation, failure, Chapter 11, renegotiation of credits) from the changes in values. The specification of a joint model for hedge fund returns and endogenous liquidation is an interesting avenue for future research.

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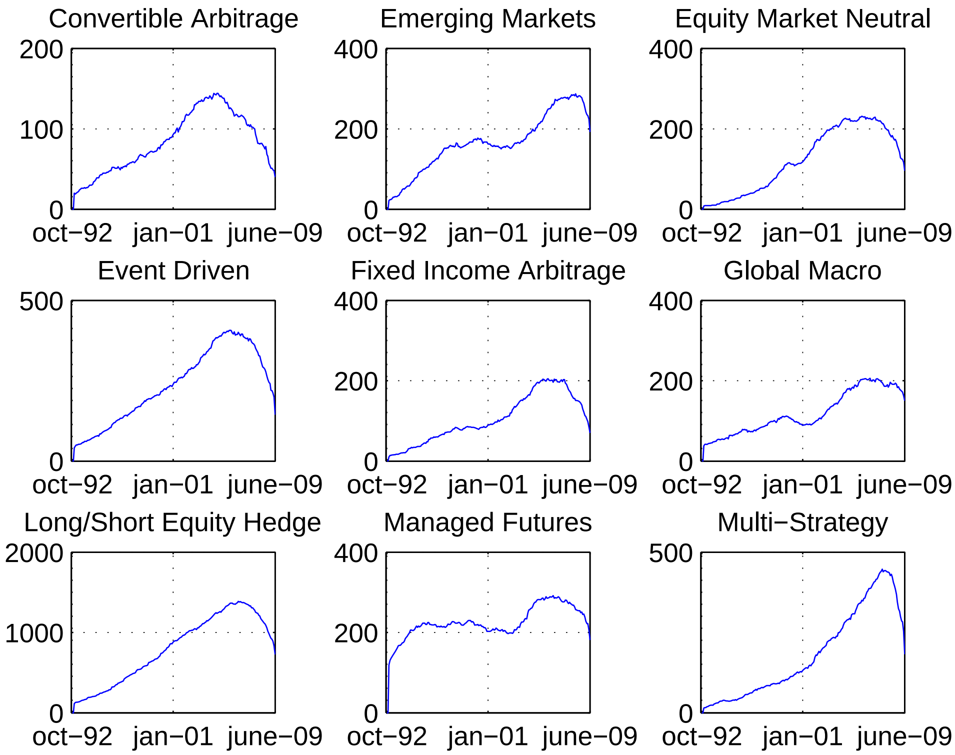
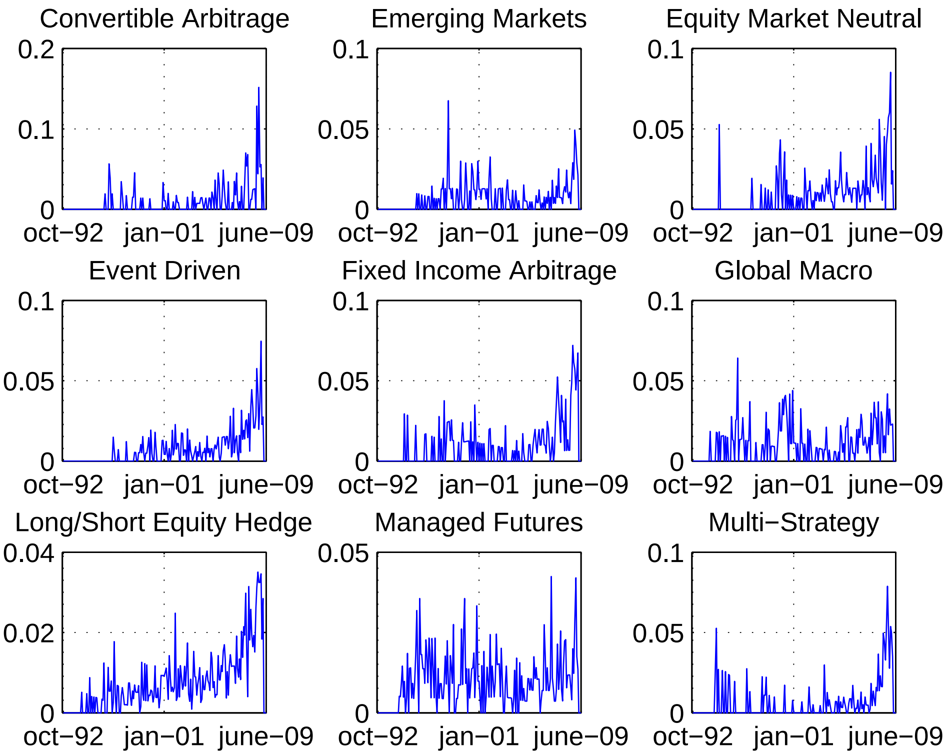
Figure 1: Subpopulation sizes of HF.



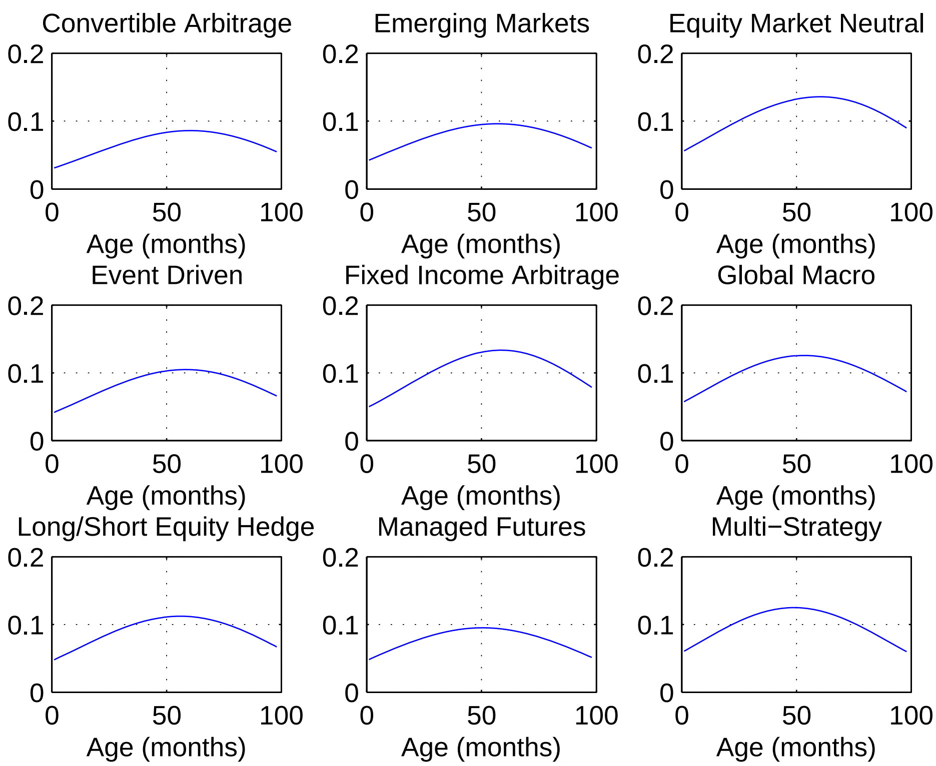
The figure displays the evolution of the population size between October 1992 and June 2009 for the nine management styles.

Figure 2: Liquidation rates of HF.



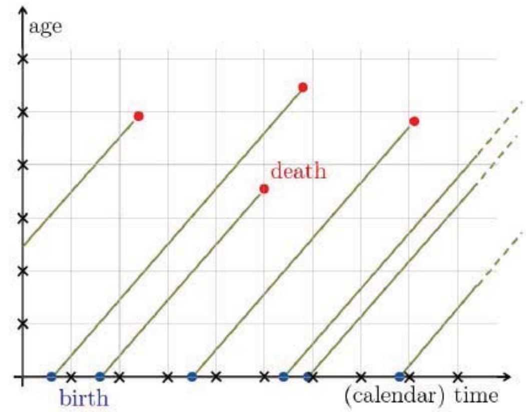
The figure displays the time series of monthly liquidation rate between October 1992 and June 2009 for the nine management styles. The liquidation rate of a management style in a given month is defined as the ratio between the liquidation count and the number of alive funds at the beginning of that month.

Figure 3: Smoothed estimates of liquidation intensity.



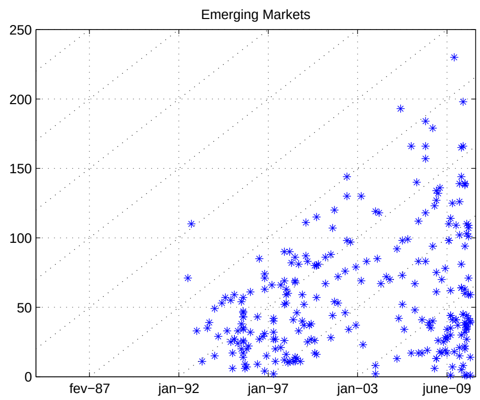
The figure displays the smoothed nonparametric estimates of the liquidation intensity as a function of age for the nine management styles. The estimates are based on the Kaplan-Meier estimators of the historical survival functions.

Figure 4: Lexis diagram.



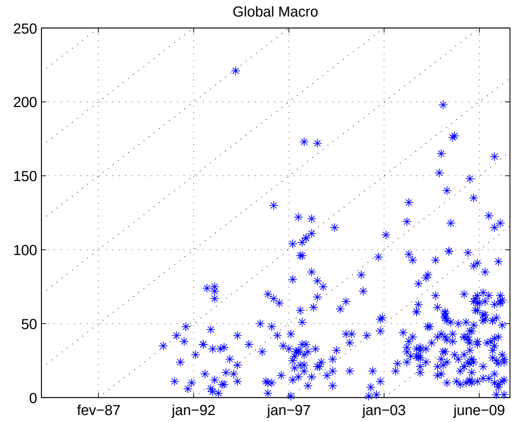
In the Lexis diagram, the liquidation of a HF is represented by a dot in the plane. The horizontal axis corresponds to the calendar time of the liquidation event, while the vertical axis displays the age of the fund at liquidation. The diagonal 45-degree lines correspond to funds in a same cohort.

Figure 5: Lexis diagram for Emerging Markets.



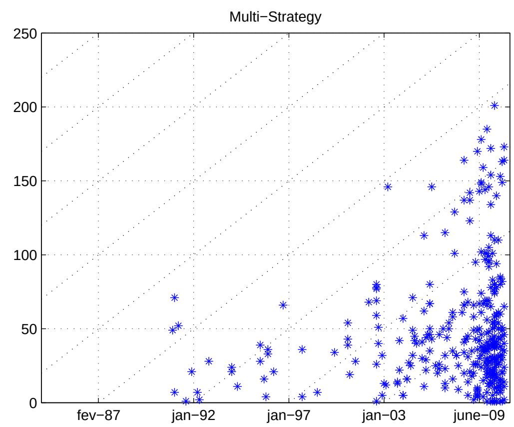
The figure displays the Lexis diagram for liquidation events of HF with management style Emerging Markets in the TASS database. The horizontal axis represents calendar time and the vertical axis represents age in months.

Figure 6: Lexis diagram for Global Macro.



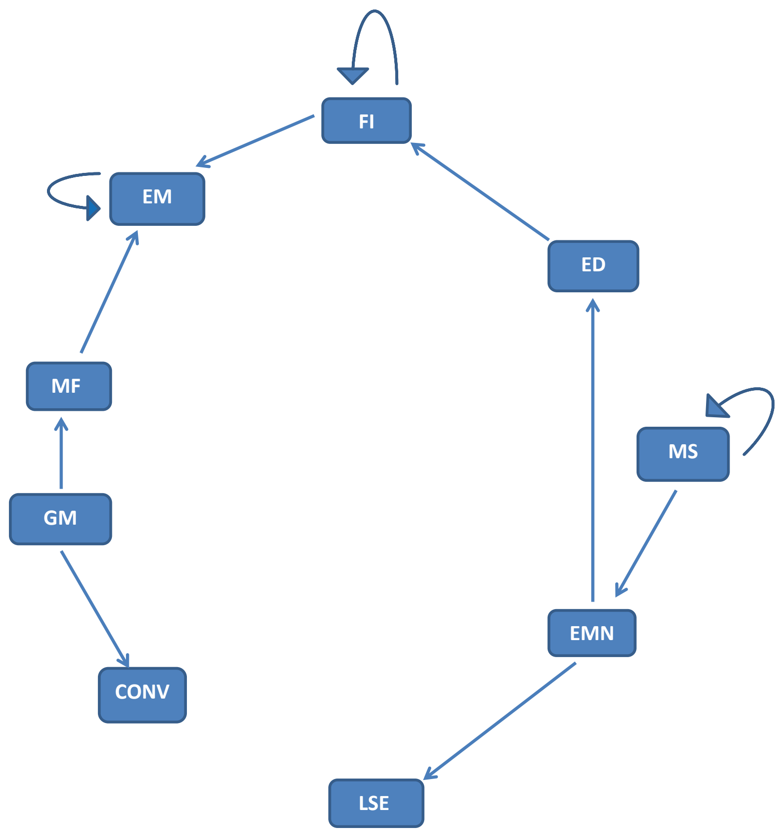
The figure displays the Lexis diagram for liquidation events of HF with management style Global Macro in the TASS database. The horizontal axis represents calendar time and the vertical axis represents age in months.

Figure 7: Lexis diagram for Multi Strategy.



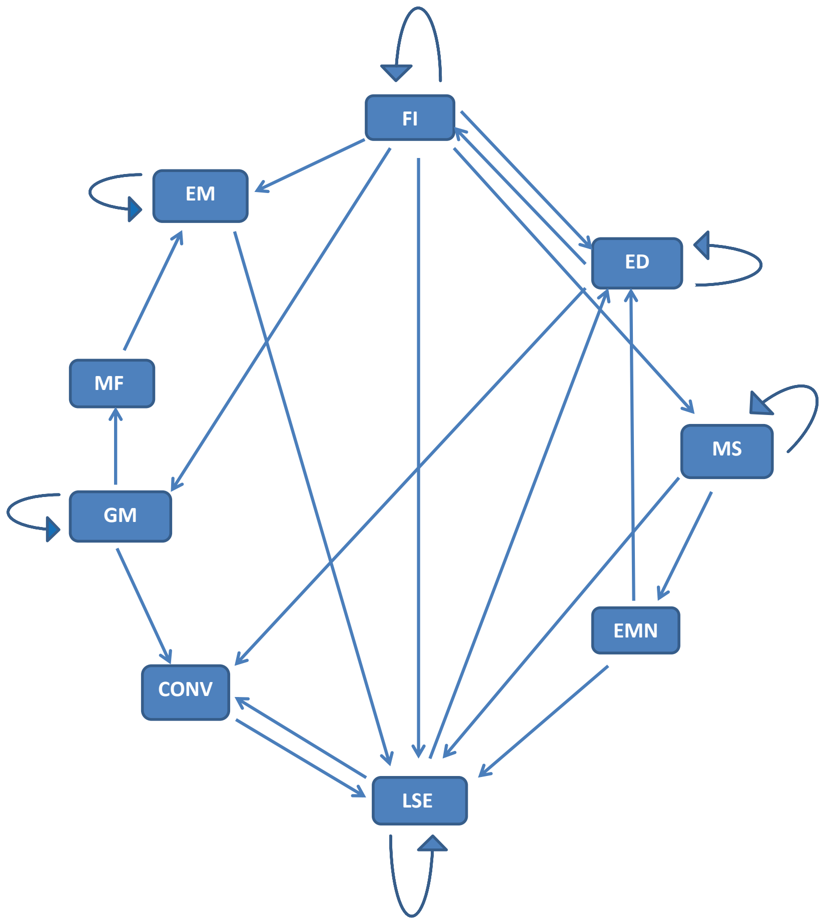
The figure displays the Lexis diagram for liquidation events of HF with management style Multi Strategy in the TASS database. The horizontal axis represents calendar time and the vertical axis represents age in months.

Figure 8: The contagion scheme for the model with contagion and frailty.



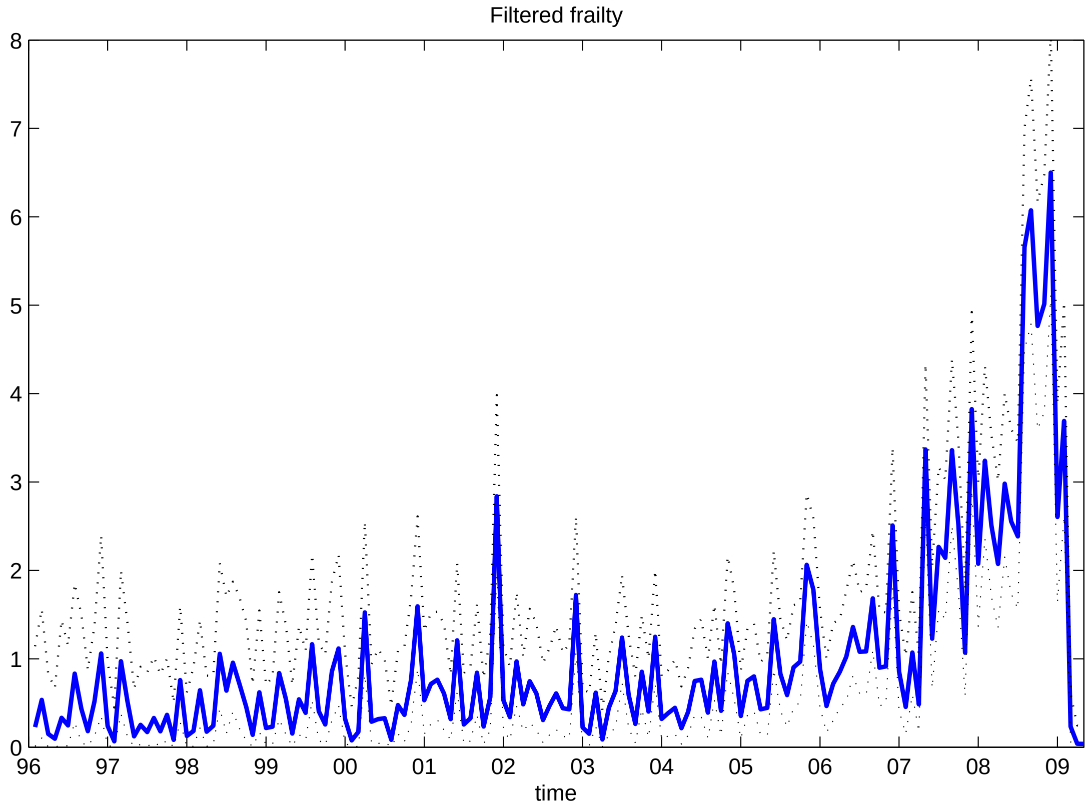
The figure provides the contagion scheme for the Poisson model with both frailty and contagion estimated on the TASS database. We display an arrow between two management styles, if the estimated contagion coefficient from the first style to the second style is statistically significant at 5% level. The estimated contagion matrix is provided in Table 3.

Figure 9: The contagion scheme for the pure contagion model.



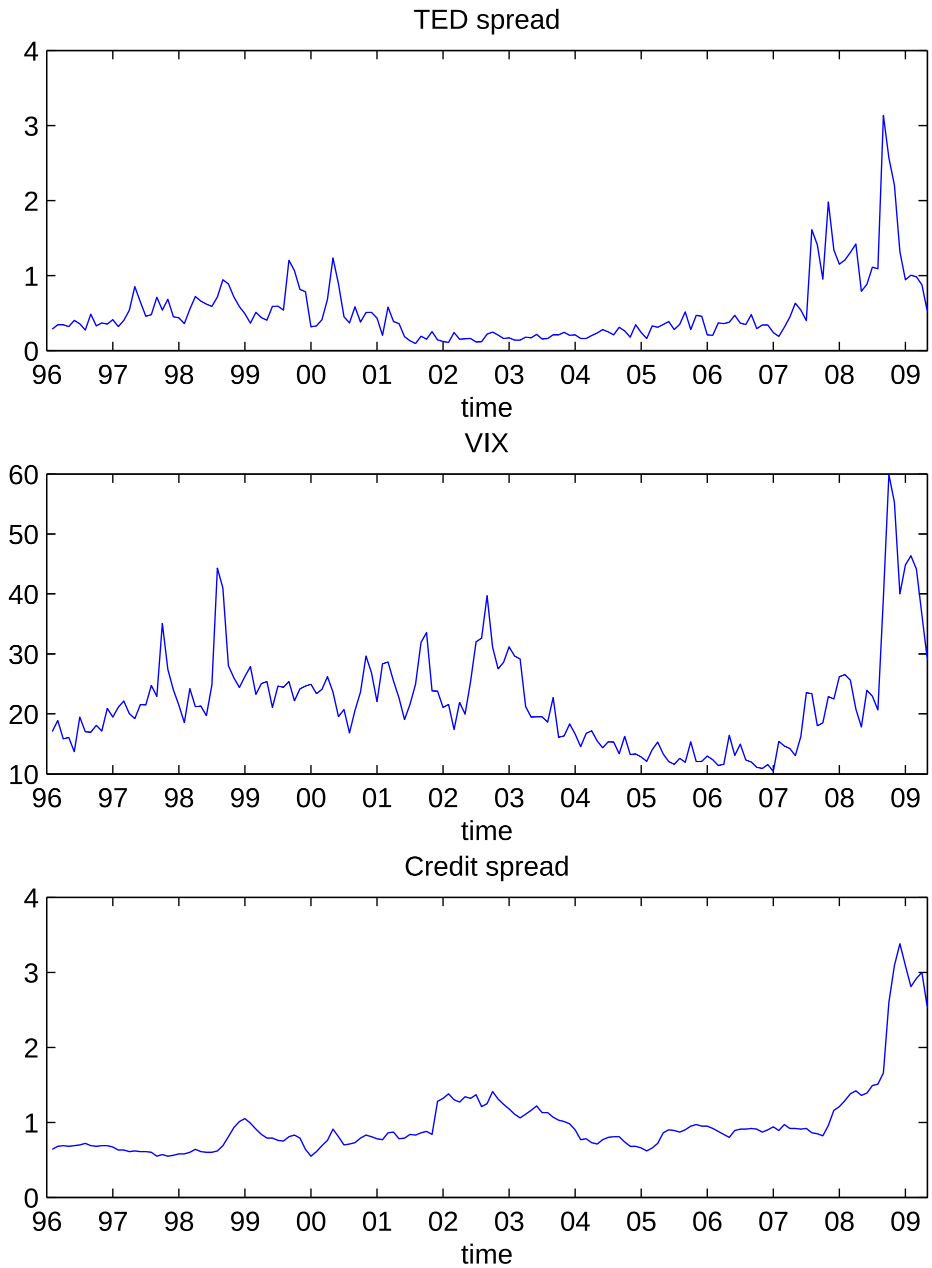
The figure provides the contagion scheme for the Poisson model with contagion only estimated on the TASS database. We display an arrow between two management styles, if the estimated contagion coefficient from the first style to the second style is statistically significant at 5% level. The estimated contagion matrix is provided in Table 5.

Figure 10: Filtered path of the frailty.



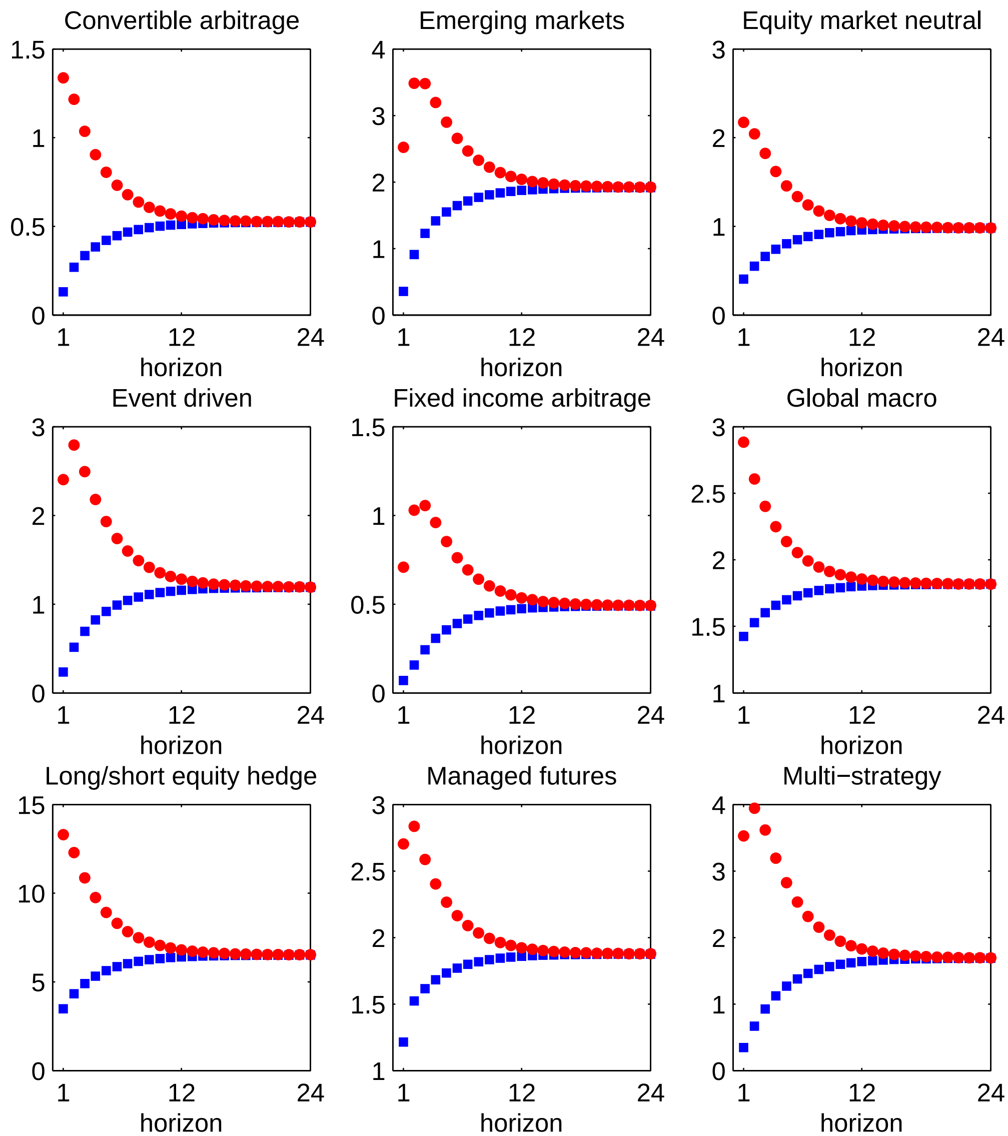
The figure displays the filtered path of the frailty (solid line) and the pointwise 95% confidence bands (dotted lines) between January 1996 and June 2009. The filtered value (resp. the lower and upper confidence bands) at a given month is the median (resp. the 2.5% and the 97.5% quantiles) of the filtering distribution of that month computed with the gamma filter described in the online supplementary material.

Figure 11: Observable liquidity indicators.



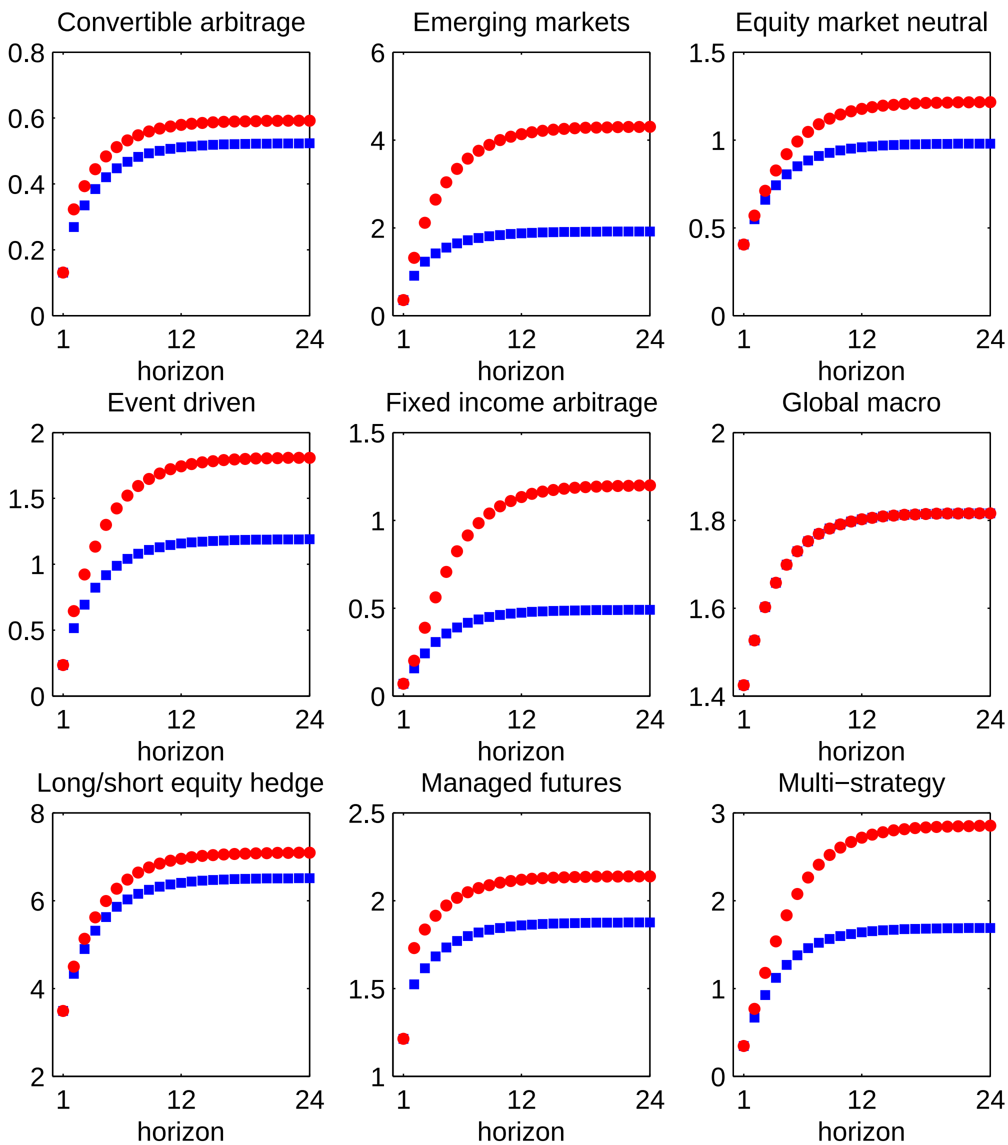
The figure displays the monthly time series of the Treasury - Eurodollar (TED) spread, the volatility index VIX, and the credit spread, measured as the difference between the BAA and AAA yields, between January 1996 and June 2009. The three series are in percentage.

Figure 12: Term structure of expected liquidation counts when stressing the current factor value.



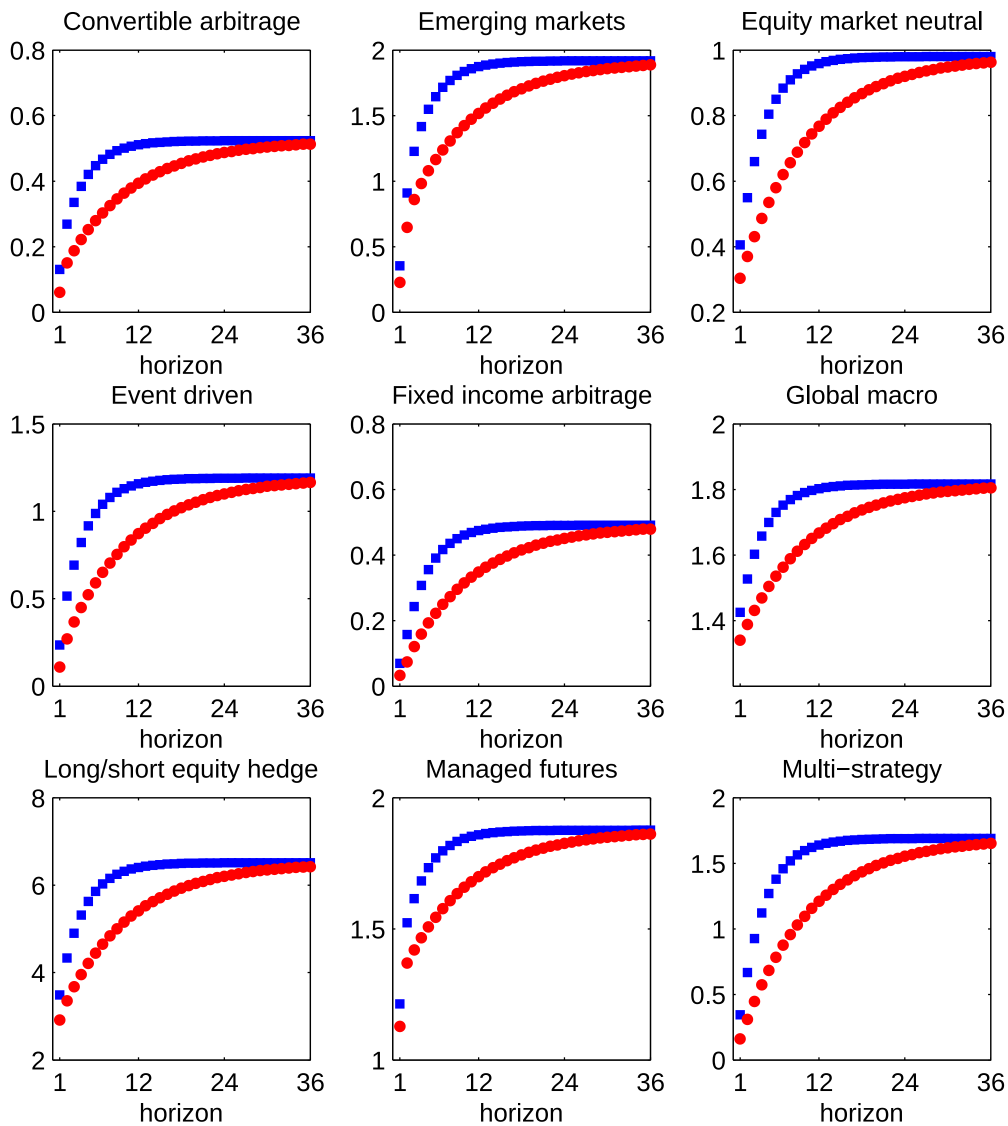
Term structure of the conditional expectation $E[Y_{k,t+\tau}|Y_t, F_t]$ of liquidation counts for horizon $\tau = 1, 2, ..., 24$ months, by management style k. Squares and circles correspond to conditioning sets with F_t equal to the filtered value of the frailty in June 2009, and the 95% quantile of the stationary distribution of the frailty, respectively. The liquidation counts vector Y_t in the conditioning set corresponds to the observations in June 2009 for both curves. The model is the specification including frailty and contagion, with intensity parameters as in Tables 2 and 3, and frailty dynamic parameters $\delta = 0.59$ and $\rho = 0.74$, corresponding to the estimates of Section 4.1.

Figure 13: Term structure of expected liquidation counts when stressing the contagion matrix.



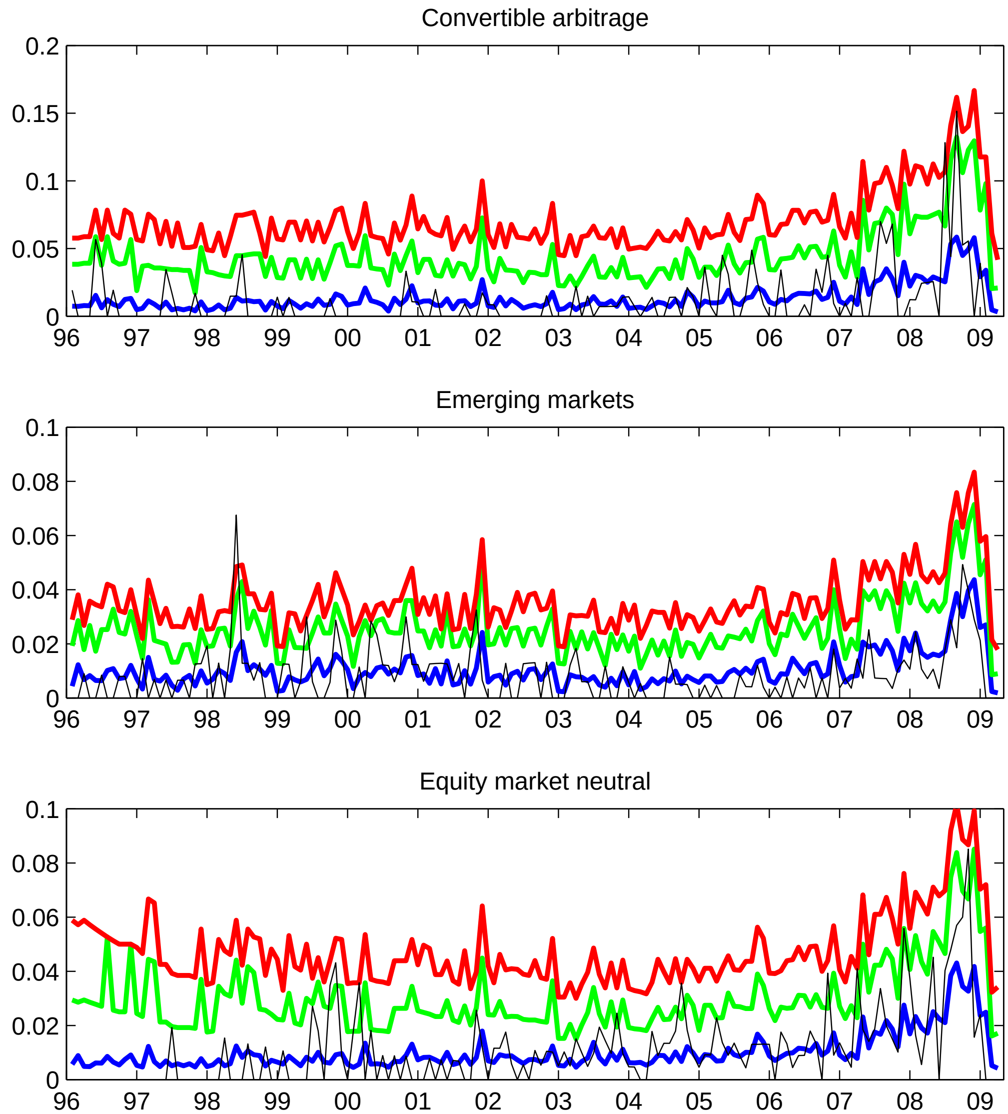
Term structure of the conditional expectation $E[Y_{k,t+\tau}|Y_t, F_t]$ of liquidation counts for horizon $\tau = 1, 2, ..., 24$ months, by management style k. Squares and circles correspond to models with contagion matrices $\hat{C}$ and $2\hat{C}$, respectively, where $\hat{C}$ is the matrix of estimates in Table 3. The intercepts and frailty sensitivities are as in Table 2, and the frailty dynamic parameters are $\delta = 0.59$ and $\rho = 0.74$, corresponding to the estimates of Section 4.1. In the conditioning set, the factor value F_t corresponds to the filtered value of the frailty in June 2009, and the liquidation counts vector Y_t corresponds to the observations in June 2009, for both curves.

Figure 14: Term structure of expected liquidation counts when stressing the frailty persistence.

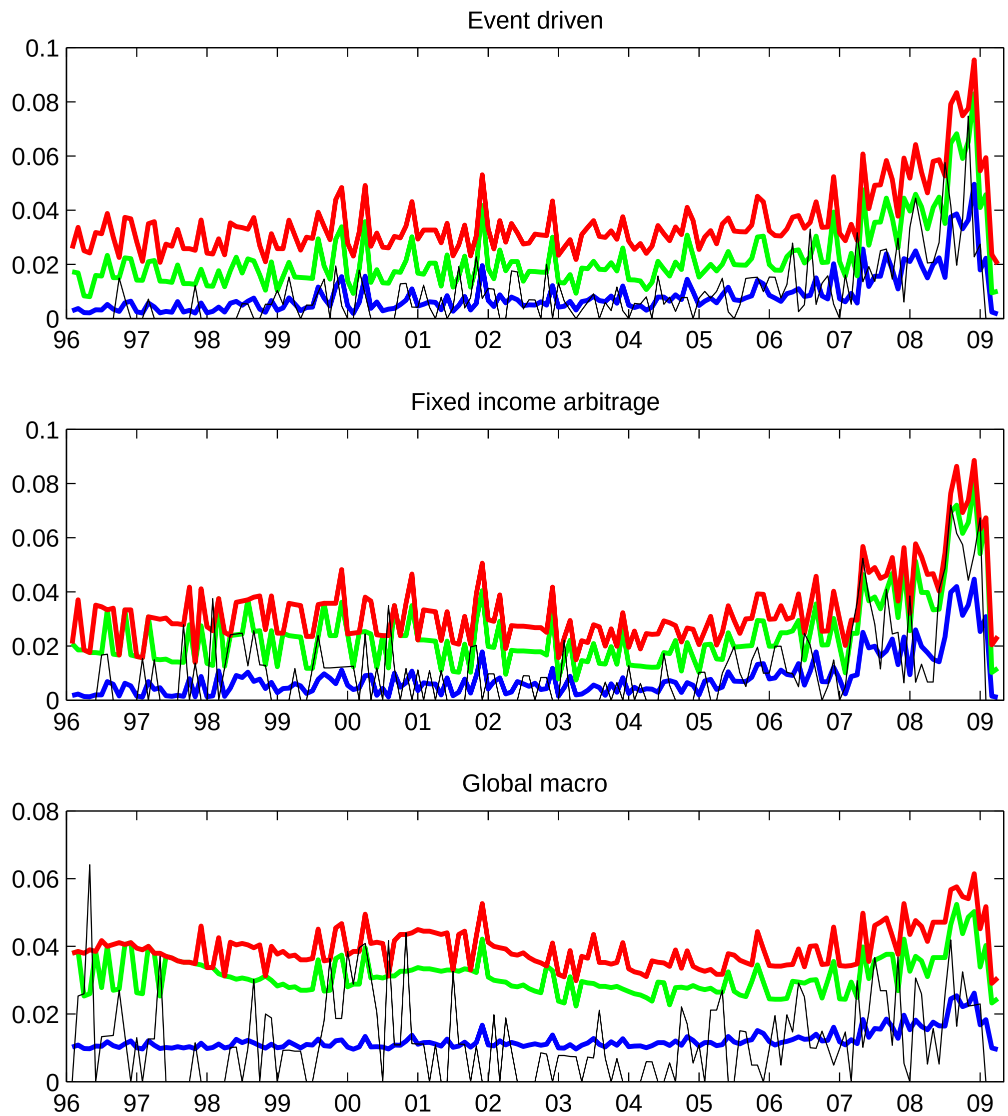


Term structure of the conditional expectation $E[Y_{k,t+\tau}|Y_t, F_t]$ of liquidation counts for horizon $\tau = 1, 2, ..., 24$ months, by management style k. Squares and circles correspond to models with frailty autocorrelation $\rho = 0.74$ (corresponding to the estimate in Section 4.1) and $\rho = 0.90$, respectively. The intensity parameters are as in Tables 2 and 3, and the parameter characterizing the stationary distribution of the frailty is $\delta = 0.59$, corresponding to the estimate of Section 4.1. In the conditioning set, the factor value F_t corresponds to the filtered value of the frailty in June 2009, and the liquidation counts vector Y_t corresponds to the observations in June 2009, for both curves.

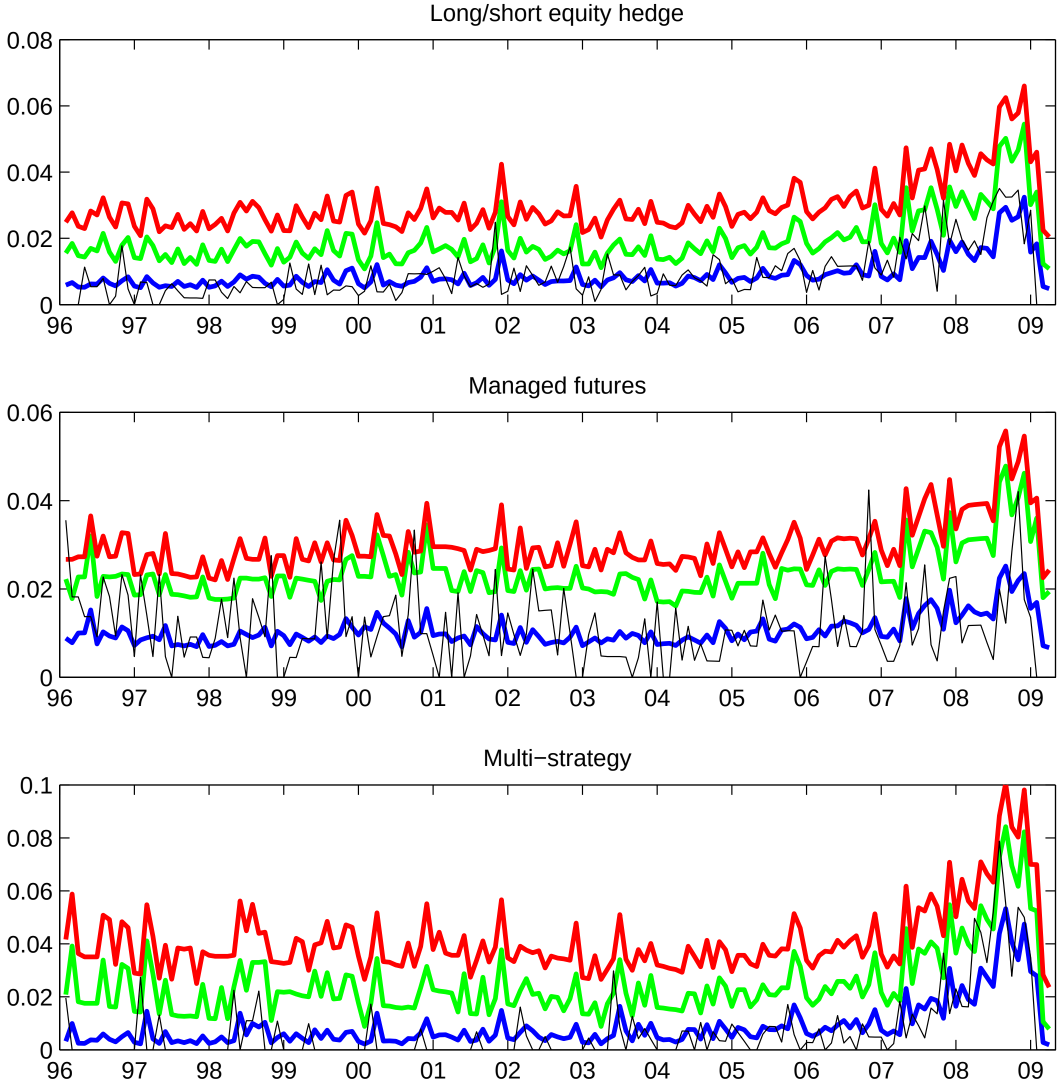
Figure 15: Time series of predictions and stress tests for liquidation rates.



In each panel, the lower thick line is the time series of the conditional expected liquidation rate $E[Y_{k,t+1}/n_{k,t+1}|Y_t, F_t]$ for a given management style in the period from January 1996 to June 2009. The factor value F_t in the conditioning information corresponds to the current filtered value $\hat{F}_t$ of the frailty, while the vector Y_t corresponds to the observed liquidation counts in the current month. The middle and upper thick lines are the time series of the conditional 95% quantile of the liquidation rate $Y_{k,t+1}/n_{k,t+1}$ in the unstressed situation, and in the stressed situation, respectively. The factor value in the conditioning information is the current filtered value $\hat{F}_t$ for the former, and the 95% quantile of the conditional distribution of the frailty given $\hat{F}_t$, for the latter. In both cases, vector Y_t in the conditioning information corresponds to the observed liquidation counts in the current month. The thin line is the time series of the realized future liquidation rate $Y_{k,t+1}/n_{k,t+1}$.



In each panel, the lower thick line is the time series of the conditional expected liquidation rate $E[Y_{k,t+1}/n_{k,t+1}|Y_t, F_t]$ for a given management style in the period from January 1996 to June 2009. The factor value F_t in the conditioning information corresponds to the current filtered value $\hat{F}_t$ of the frailty, while the vector Y_t corresponds to the observed liquidation counts in the current month. The middle and upper thick lines are the time series of the conditional 95% quantile of the liquidation rate $Y_{k,t+1}/n_{k,t+1}$ in the unstressed situation, and in the stressed situation, respectively. The factor value in the conditioning information is the current filtered value $\hat{F}_t$ for the former, and the 95% quantile of the conditional distribution of the frailty given $\hat{F}_t$, for the latter. In both cases, vector Y_t in the conditioning information corresponds to the observed liquidation counts in the current month. The thin line is the time series of the realized future liquidation rate $Y_{k,t+1}/n_{k,t+1}$.



In each panel, the lower thick line is the time series of the conditional expected liquidation rate $E[Y_{k,t+1}/n_{k,t+1}|Y_t, F_t]$ for a given management style in the period from January 1996 to June 2009. The factor value F_t in the conditioning information corresponds to the current filtered value $\hat{F}_t$ of the frailty, while the vector Y_t corresponds to the observed liquidation counts in the current month. The middle and upper thick lines are the time series of the conditional 95% quantile of the liquidation rate $Y_{k,t+1}/n_{k,t+1}$ in the unstressed situation, and in the stressed situation, respectively. The factor value in the conditioning information is the current filtered value $\hat{F}_t$ for the former, and the 95% quantile of the conditional distribution of the frailty given $\hat{F}_t$, for the latter. In both cases, vector Y_t in the conditioning information corresponds to the observed liquidation counts in the current month. The thin line is the time series of the realized future liquidation rate $Y_{k,t+1}/n_{k,t+1}$.

Table 1: The database.

		Alive funds	(%)	Liquidated funds	(%)	**Total**	**(%)**
CONV	Convertible Arbitrage	45	2.00%	66	4.30%	**111**	**2.90%**
EM	Emerging Markets	227	10.00%	111	7.30%	**338**	**8.90%**
EMN	Equity Market Neutral	126	5.50%	139	9.10%	**265**	**7.00%**
ED	Event Driven	216	9.50%	129	8.50%	**345**	**9.10%**
FI	Fixed Income Arbitrage	95	4.20%	75	4.90%	**170**	**4.50%**
GM	Global Macro	162	7.10%	102	6.70%	**264**	**6.90%**
LSE	Long/Short Equity Hedge	885	38.80%	546	35.90%	**1431**	**37.70%**
MF	Managed Futures	224	9.80%	230	15.10%	**454**	**12.00%**
MS	Multi-Strategy	299	13.10%	122	8.00 %	**421**	**11.10%**
	Total	**2279**	**100.00%**	**1520**	**100.00%**	**3799**	**100.00%**

The table provides the distribution of alive funds on June 2009, and funds liquidated prior to June 2009, across the nine management styles.

Table 2: Estimated intercepts and factor sensitivities in the model with contagion and frailty.

	Intercept a_k	Sensitivity b_k
Convertible Arbitrage	0.00	1.08**
	(0.15)	*(0.55)*
Emerging Markets	0.10	0.69**
	(0.23)	*(0.27)*
Equity Market Neutral	0.27	0.84**
	(0.30)	*(0.40)*
Event Driven	0.00	1.39**
	(0.15)	*(0.70)*
Fixed Income Arbitrage	0.00	0.31**
	(0.20)	*(0.13)*
Global Macro	0.76***	0.33***
	(0.14)	*(0.12)*
Long/Short Equity Hedge	2.98***	4.55**
	(1.10)	*(1.92)*
Managed Futures	1.18	0.63**
	(0.75)	*(0.25)*
Multi-Strategy	0.00	0.89**
	(0.17)	*(0.41)*

The table provides the estimates of the intercepts a_k and frailty sensitivities b_k for the Poisson model with frailty and contagion:

$$Y_{k,t} \sim \mathcal{P}\left[\gamma_{k,t}(a_k + b_k F_t + c_k' Y_{t-1}^*)\right], \quad k = 1, ..., 9,$$

where $Y_{k,t}$ is the liquidation count for management style k in month t, the vector Y_{t-1}^* gathers the lagged liquidation frequencies in the nine management styles, and F_t is the unobservable frailty. The vector c_k contains the contagion coefficients for management style k. The adjustment term $\gamma_{k,t} = n_{k,t}/n_{k,t_0}$ scaling the liquidation intensity accounts for the time-varying sizes $n_{k,t}$ of the classes, and the reference date t_0 is January 1999. The sample period for estimation is from January 1996 to June 2009. The estimates are obtained by the Generalized Method of Moments (GMM) estimator of Appendix B. Standard errors are provided in parentheses. Stars $*$, $**$ and $***$ denote significance at the 10%, 5% and 1% level, respectively.

Table 3: Estimated contagion parameters in the model with contagion and frailty.

	Convertible Arbitrage	Emerging Markets	Equity Market Neutral	Event Driven	Fixed Income Arbitrage	Global Macro	Long/Short Equity Hedge	Managed Futures	Multi-Strategy
Convertible Arbitrage						0.15**			
						(0.07)			
Emerging Markets		0.17***			0.21**			0.20***	
		(0.06)			*(0.08)*			*(0.05)*	
Equity Market Neutral									0.10**
									(0.05)
Event Driven			0.35***	0.10*					
			(0.09)	*(0.06)*					
Fixed Income Arbitrage				0.09**	0.22***				
				(0.04)	*(0.07)*				
Global Macro					0.20*				
					(0.11)				
Long/Short Equity Hedge			0.39**						
			(0.16)						
Managed Futures						0.27**			
						(0.11)			
Multi-Strategy									0.29***
									(0.06)

The table provides the estimates of the contagion parameters $(c_{k,l})$ for the Poisson model with frailty and contagion. Rows and columns correspond to target and source of contagion, respectively. The Poisson model with frailty and contagion is:

$$Y_{k,t} \sim \mathcal{P}\left[\gamma_{k,t}(a_k + b_k F_t + c_k' Y_{t-1}^*)\right], \quad k = 1, ..., 9,$$

where $Y_{k,t}$ is the liquidation count for management style k in month t, the vector Y_{t-1}^* gathers the lagged liquidation frequencies in the nine management styles, and F_t is the unobservable frailty. The coefficient b_k is the frailty sensitivity of management style k. The vector c_k contains the contagion coefficients $c_{k,1}, ..., c_{k,9}$ for management style k, corresponding to the k-th row in the table. The adjustment term $\gamma_{k,t} = n_{k,t}/n_{k,t_0}$ scaling the liquidation intensity accounts for the time-varying sizes $n_{k,t}$ of the classes, and the reference date t_0 is January 1999. The sample period for estimation is from January 1996 to June 2009. The parameters are estimated by the Generalized Method of Moments (GMM) estimator of Appendix B. Standard errors are provided in parentheses. Stars $*$, $**$ and $***$ denote significance at the 10%, 5% and 1% level, respectively. The estimates that are not statistically significant at 10% level are not displayed.

Table 4: Estimated intercepts in the pure contagion model.

	Intercept a_k
Convertible Arbitrage	0.04
	(0.14)
Emerging Markets	0.40***
	(0.15)
Equity Market Neutral	0.60***
	(0.18)
Event Driven	0.45**
	(0.21)
Fixed Income Arbitrage	0.23*
	(0.12)
Global Macro	0.51***
	(0.13)
Long/Short Equity Hedge	4.17***
	(0.48)
Managed Futures	1.57***
	(0.23)
Multi-Strategy	0.21*
	(0.11)

The table provides the Maximum Likelihood (ML) estimates of the intercept parameters a_k for the Poisson model with contagion only:

$$Y_{k,t} \sim \mathcal{P}\left[\gamma_{k,t}(a_k + c_k' Y_{t-1}^*)\right], \quad k = 1, ..., 9,$$

where $Y_{k,t}$ is the liquidation count for management style k in month t, and the vector Y_{t-1}^* gathers the lagged liquidation frequencies in the nine management styles. The vector c_k contains the contagion coefficients for management style k. The adjustment term $\gamma_{k,t} = n_{k,t}/n_{k,t_0}$ scaling the liquidation intensity accounts for the time-varying sizes $n_{k,t}$ of the classes, and the reference date t_0 is January 1999. The sample period for estimation is from January 1996 to June 2009. Standard errors are provided in parentheses. Stars $*$, $**$ and $***$ denote significance at the 10%, 5% and 1% level, respectively.

Table 5: Estimated contagion parameters in the pure contagion model.

	Convertible Arbitrage	Emerging Markets	Equity Market Neutral	Event Driven	Fixed Income Arbitrage	Global Macro	Long/Short Equity Hedge	Managed Futures	Multi-Strategy
Convertible Arbitrage				0.11** *(0.05)*		0.17** *(0.08)*	0.05** *(0.02)*		
Emerging Markets		0.16** *(0.07)*			0.28*** *(0.09)*			0.15*** *(0.05)*	
Equity Market Neutral									0.24*** *(0.10)*
Event Driven			0.35*** *(0.11)*	0.15** *(0.07)*	0.31** *(0.14)*		0.08*** *(0.03)*		
Fixed Income Arbitrage				0.08** *(0.04)*	0.30*** *(0.08)*				
Global Macro					0.15** *(0.08)*	0.22*** *(0.07)*			
Long/Short Equity Hedge	0.45*** *(0.16)*	0.19** *(0.09)*	0.42** *(0.20)*		0.64*** *(0.25)*		0.19*** *(0.06)*		0.82*** *(0.25)*
Managed Futures						0.27** *(0.12)*			
Multi-Strategy					0.12** *(0.06)*				0.44*** *(0.07)*

The table provides the Maximum Likelihood (ML) estimates of the contagion parameters $(c_{k,l})$ for the Poisson model with contagion only. Rows and columns correspond to target and source of contagion, respectively. The Poisson model with pure contagion is:

$$Y_{k,t} \sim \mathcal{P}\left[\gamma_{k,t}(a_k + c_k' Y_{t-1}^*)\right], \quad k = 1, ..., 9,$$

where $Y_{k,t}$ is the liquidation count for management style k in month t, and the vector Y_{t-1}^* gathers the lagged liquidation frequencies in the nine management styles. The vector c_k contains the contagion coefficients $c_{k,1}, ..., c_{k,9}$ for management style k, corresponding to the k-th row in the table. The adjustment term $\gamma_{k,t} = n_{k,t}/n_{k,t_0}$ scaling the liquidation intensity accounts for the time-varying sizes $n_{k,t}$ of the classes, and the reference date t_0 is January 1999. The sample period for estimation is from January 1996 to June 2009. Standard errors are provided in parentheses. Stars $**$ and $***$ denote significance at the 5% and 1% level, respectively. The estimates that are not statistically significant at 5% level are not displayed.

Table 6: Decomposition of the variance.

	Underlying Poisson	Contagion	Frailty (direct effect)	Frailty (propagated by contagion)
Percentage of variance	6.54 %	5.10 %	64.30 %	24.06 %

The table provides the decomposition of the variance of the payoff of an equally weighted portfolio of liquidation swaps written on the individual hedge funds in our dataset. The liquidation swap for management style k pays 1 USD for each fund of style k that is liquidated in a given month. The decomposition involves: *i)* the variance of the payoff due to the underlying Poisson shocks, if the effects of both frailty and contagion were zero, *ii)* the contribution from pure contagion (not accounting for frailty), *iii)* the direct effect of frailty (not accounting for contagion), and *iv)* the indirect effect of frailty, propagated by contagion. The variance decomposition is computed using the estimated Poisson model with frailty and contagion (see Tables 2 and 3).

Table 7: Redemption frequency, leverage and factor sensitivity.

	Red. freq. ≤ 1 m	1 m $<$ Red. freq. ≤ 3 m	Red. freq. > 3 m	Leverage	Sensitivity b_k
Convertible Arbitrage	49%	47%	4%	77%	1.08
Emerging Markets	66%	31%	3%	57%	0.69
Equity Market Neutral	68%	28%	4%	58%	0.84
Event Driven	36%	48%	15%	54%	1.39
Fixed Income Arbitrage	46%	48%	6%	68%	0.31
Global Macro	80%	18%	2%	70%	0.33
Long/Short Equity Hedge	56%	37%	7%	57%	4.55
Managed Futures	92%	7%	1%	79%	0.63
Multi-Strategy	67%	29%	4%	50%	0.89

The second, third and fourth columns display the percentages of hedge funds with redemption frequency smaller or equal to 1 month, between 1 month and 3 months, and larger than 3 months, respectively, for the nine management styles. The fifth column displays the percentage of hedge funds reporting some use of leverage. The information concerns only whether leverage is used or not, and not its amount. For comparison purpose, the last column of the table displays the sensitivities to the frailty estimated by GMM in the Poisson model with frailty and contagion (see Table 2).

Table 8: Regression of the frailty on observable variables.

Model	$Const$	TED	$TEDL$	VIX	$VIXL$	SPR							R^2
1	0.18 (0.10)	1.63*** (0.15)											0.44
2	− 0.05 (0.11)	1.02*** (0.22)	0.88*** (0.24)										0.49
3	0.24 (0.25)			0.04*** (0.01)									0.08
4	1.43*** (0.32)			0.08*** (0.01)	−0.10*** (0.02)								0.22
5	1.00*** (0.26)	0.82*** (0.23)	1.07*** (0.24)	0.01 (0.01)	−0.06*** (0.02)								0.53
6	− 0.11 (0.16)					1.16*** (0.15)							0.28
7	0.94*** (0.24)	0.95*** (0.21)	0.49** (0.24)	− 0.00 (0.02)	−0.07*** (0.02)	0.92*** (0.16)							0.62
	$I(VIX \leq c)$						$I(VIX > c)$						
Model	$Const$	TED	$TEDL$	VIX	$VIXL$	SPR	$Const$	TED	$TEDL$	VIX	$VIXL$	SPR	
8	1.37*** (0.33)	0.82** (0.36)	0.86** (0.41)	− 0.05 (0.03)	− 0.02 (0.02)		1.18 (0.85)	0.86** (0.30)	1.26*** (0.30)	0.03 (0.02)	−0.09*** (0.03)		0.57
9	− 0.32 (0.36)	0.49* (0.30)	0.81** (0.34)	0.00 (0.03)	−0.06*** (0.02)	2.10*** (0.29)	3.31*** (0.98)	0.88*** (0.25)	0.57* (0.34)	0.01 (0.02)	−0.18*** (0.04)	0.99*** (0.31)	0.70

The table displays the estimated coefficients in the regression of the filtered frailty on some sets of observed variables, which include: the constant (Const), the Treasury - Eurodollar (TED) spread, the average value of the TED spread over the previous quarter (TEDL), the volatility index (VIX), the average of the VIX over the previous 12 months (VIXL), and the credit spread (SPR), measured as the difference between the BAA and AAA yields. The regression coefficients can be different according to whether the VIX is below, or above, a threshold c. The estimated threshold is $c = 25$ (monthly percentage value). Model 9 corresponds to the switching regression:

$$\begin{aligned}\hat{F}_t &= I(VIX_t \leq c)\,(\beta_1 + \beta_2 TED_t + \beta_3 TEDL_t + \beta_4 VIX_t + \beta_5 VIXL_t + \beta_6 SPR_t) \\ &\quad + I(VIX_t > c)\,(\gamma_1 + \gamma_2 TED_t + \gamma_3 TEDL_t + \gamma_4 VIX_t + \gamma_5 VIXL_t + \gamma_6 SPR_t) + e_t,\end{aligned}$$

where the explained variable $\hat{F}_t$ is the filtered frailty, while Models 1-8 are constrained specifications. Standard errors are given in parentheses. Stars $*$, $**$ and $***$ denote significance at the 10%, 5% and 1% level, respectively. The last column provides the R^2 of the regressions.

Appendix A: The Poisson model with contagion and frailty

In this Appendix we analyze in more depth the Poisson model with contagion and frailty defined in Section 2. We focus on the Autoregressive Gamma specification for the frailty dynamics (Section A.1) and the affine property of the joint process of liquidation counts and frailty (Section A.2), we derive the stationarity conditions for this joint process (Section A.3) and its unconditional moments (Section A.4).

A.1. The Autoregressive Gamma (ARG) process

In this section we review the main properties of the ARG(1) process [see Gourieroux, Jasiak (2006)].

i) The conditional distribution

The ARG(1) process (F_t) is a Markov process with conditional distribution the noncentral gamma distribution $\gamma(\delta, \eta F_{t-1}, \nu)$, where δ, $\delta > 0$, is the degree of freedom, ηF_{t-1}, $\eta > 0$, the noncentrality parameter and ν, $\nu > 0$, a scale parameter. Its first- and second-order conditional moments are:

$$E(F_t|F_{t-1}) = \delta\nu + \eta\nu F_{t-1}, \quad V(F_t|F_{t-1}) = \nu^2\delta + 2\eta\nu^2 F_{t-1}. \tag{a.1}$$

The ARG(1) process is a discrete-time affine process, that is, the conditional Laplace transform is an exponential affine function of the lagged variable:

$$E[\exp(-uF_t)|F_{t-1}] = \exp[-\alpha(u)F_{t-1} - \beta(u)], \tag{a.2}$$

where functions α and β are given by:

$$\alpha(u) = \frac{\eta\nu u}{1+\nu u}, \quad \beta(u) = \delta \log(1+\nu u), \tag{a.3}$$

for any real value of the argument $u > -1/\nu$.

ii) The state space representation

The ARG(1) process admits a state space representation, which is especially convenient for simulating the paths of the process. To get a simulated value of F_t given F_{t-1}, we proceed as follows:
a) We draw an intermediate value Z_t^s in a Poisson distribution $\mathcal{P}(\eta F_{t-1})$;
b) Then, F_t is drawn in the centered gamma distribution $\gamma(\delta + Z_t^s, 0, \nu)$.

iii) Stationarity condition and stationary distribution

The ARG(1) process is stationary if $\rho = \nu\eta$ is such that $\rho < 1$. Then, the stationary distribution is a centered gamma distribution $\gamma(\delta, 0, \frac{\nu}{1-\nu\eta})$. In particular, we get the unconditional moments:

$$E(F_t) = \frac{\nu\delta}{1-\nu\eta}, \quad V(F_t) = \delta\left(\frac{\nu}{1-\nu\eta}\right)^2. \tag{a.4}$$

From the first equation in (a.1) it is seen that parameter ρ is the first-order autocorrelation of process (F_t).

iv) Normalization and reparameterization

When the $ARG(1)$ process is used as a latent frailty, the scale of the process can be absorbed in the sensitivity parameters b_k of the intensity function in (2.1). Then, the process (F_t) can be normalized to have stationary expectation $E(F_t) = 1$. Thus, from the first equation in (a.4), the parameters are such that $\nu\delta = 1 - \nu\eta = 1 - \rho$. It follows that the model can be parameterized in terms of δ and ρ, while the remaining parameters are given by:

$$\nu = \frac{1-\rho}{\delta}, \qquad \eta = \frac{\rho\delta}{1-\rho}. \tag{a.5}$$

The stationary distribution is $\gamma(\delta, 0, 1/\delta)$, with Laplace transform $E[\exp(-uF_t)] = (1 + u/\delta)^{-\delta}$, for $u > -\delta$. Moreover, the stationary variance is $V(F_t) = 1/\delta$.

A.2 The affine property

In this section we show that the joint process (Y_t, F_t) of liquidation counts and frailty defined in Assumptions A.1-A.2 (Section 2) is affine. By the moment generating function of the Poisson distribution, we deduce that the conditional Laplace transform of the current liquidation counts vector Y_t given $\underline{F_t}$ and $\underline{Y_{t-1}}$ is:

$$\begin{aligned}
\psi_t(u) &= E\left[\exp(-u'Y_t)|\underline{F_t}, \underline{Y_{t-1}}\right] = \prod_k E[\exp(-u_k Y_{k,t})|\underline{F_t}, \underline{Y_{t-1}}] \\
&= \prod_k \exp\{-\gamma_{k,t}\lambda_{k,t}[1 - \exp(-u_k)]\} \\
&= \exp\left\{-\sum_k [1 - \exp(-u_k)]\gamma_{k,t}a_k - \sum_k [1 - \exp(-u_k)]\gamma_{k,t}b_k F_t - \sum_k [1 - \exp(-u_k)]\gamma_{k,t}c_k' Y_{t-1}^*\right\},
\end{aligned} \tag{a.6}$$

where u is a vector with nonnegative components u_k, and we use the notation $\lambda_{k,t} = (n_{k,t}/n_{k,t_0})(a_k + b_k F_t + c_k' Y_{t-1}^*)$ for the conditional intensity and $\gamma_{k,t} = n_{k,t}/n_{k,t_0}$ for the size adjustment in management style k. Moreover, by the exogenous ARG(1) dynamics of the frailty process, we have:

$$E[\exp(-vF_t)|\underline{Y_{t-1}}, \underline{F_{t-1}}] = \exp[-\alpha(v)F_{t-1} - \beta(v)], \tag{a.7}$$

where functions α and β are given in (a.3). Then, from (a.6)-(a.7) and the Law of iterated expectation, we get the conditional Laplace transform of the joint process $(Y_t', F_t)'$ given its past:

$$\begin{aligned}
\psi_t(u, v) &= E[\exp(-u'Y_t - vF_t)|\underline{F_{t-1}}, \underline{Y_{t-1}}] \\
&= E\left\{E\left[\exp(-u'Y_t)|\underline{F_t}, \underline{Y_{t-1}}\right]\exp(-vF_t)|\underline{F_{t-1}}, \underline{Y_{t-1}}\right\} \\
&= \exp\left\{-\sum_k \gamma_{k,t}a_k[1 - \exp(-u_k)] - \sum_k [1 - \exp(-u_k)]\gamma_{k,t}c_k' Y_{t-1}^*\right. \\
&\qquad \left. -\alpha\left(\sum_{k,} \gamma_{k,t}b_k[1 - \exp(-u_k)] + v\right)F_{t-1} - \beta\left(\sum_k \gamma_{k,t}b_k[1 - \exp(-u_k)] + v\right)\right\}.
\end{aligned} \tag{a.8}$$

This joint conditional Laplace transform is exponential affine in lagged values Y_{t-1} and F_{t-1}, that is, it is of the form:

$$\psi_t(u,v) = \exp\{-\alpha_{1,t}(u,v)'Y_{t-1} - \alpha_{2,t}(u,v)F_{t-1} - \beta_t(u,v)\}, \text{ say,} \tag{a.9}$$

where the sensitivity coefficients $\alpha_{1,t}$, $\alpha_{2,t}$ and β_t depend on the class sizes and are time dependent in general. Thus, process $(Y_t', F_t)'$ is a time-heterogeneous affine Markov process. The closed form exponential affine expression of the conditional Laplace transform of process $(Y_t', F_t)'$ simplifies the computation of the predictive distributions at any prediction horizon required in the stress-testing analysis, and the filtering of the latent factor (see the online supplementary material). The closed form expression of the conditional Laplace transform also provides informative moment restrictions, which are the basis of estimation with the Generalized Method of Moments (see Appendix B). While we have kept the frailty process one-dimensional with ARG dynamics for simplicity, the affine property of the joint process $(Y_t', F_t)'$ remains valid as long as the frailty admits a (multidimensional) affine dynamics.

A.3 Stationarity

In some applications such as stress testing (see Section 6), it is appropriate to consider a given portfolio structure with respect to the management style, which is held fixed through time in the analysis. For this purpose, we consider the next Assumption A.3 in the rest of this Appendix.

Assumption A.3: *The size adjustments are constant and equal to* 1*:* $\gamma_{k,t} = 1$*, for any* k, t*.*

Thus, the sizes of the categories are fixed through time, and are assumed homogeneous across management styles for expository purpose.

Let us now derive the stationarity conditions for the joint process $Z_t = (Y_t', F_t)'$ under Assumptions A.1-A.3. From equation (a.8), the conditional Laplace transform of the Markov process Z_t is given by:

$$E\left[\exp\left(-w'Z_t\right)|Z_{t-1}\right] = \exp\left(-A(w)'Z_{t-1} - B(w)\right),$$

where $w = (u', v)' \in \mathbb{R}^K \times \mathbb{R}$, functions $A(w)$ and $B(w)$ are given by:

$$A(w) = \left[\sum_{k=1}^{K} c_k'(1-e^{-u_k}), \alpha\left(v + \sum_{k=1}^{K}(1-e^{-u_k})b_k\right)\right]',$$

and:

$$B(w) = \sum_{k=1}^{K}(1-e^{-u_k})a_k + \beta\left(v + \sum_{k=1}^{K}(1-e^{-u_k})b_k\right),$$

and functions α and β are given in (a.3). Thus, process (Z_t) is a time-homogeneous affine process. From Proposition 2 in Darolles, Gourieroux, Jasiak (2006), process (Z_t) is strictly stationary if:

$$\lim_{\tau\to\infty}\left[\frac{\partial A(0)}{\partial w'}\right]^{\tau} = 0. \tag{a.10}$$

Now, by using that $\dfrac{\partial A(0)}{\partial u_k} = [c_k', b_k d\alpha(0)/du]' = [c_k, \rho b_k]'$, for $k = 1, ..., K$, and $\dfrac{\partial A(0)}{\partial v} = [0', d\alpha(0)/du]' =$

$[0', \rho]'$, we get:

$$\frac{\partial A(0)}{\partial w'} = \begin{pmatrix} C & 0 \\ \rho b' & \rho \end{pmatrix}.$$

Thus, condition (a.10) is satisfied if, and only if, the first-order autocorrelation of the frailty is such that $\rho < 1$ and the eigenvalues of the contagion matrix C have modulus smaller than 1.

A.4 Unconditional moments (Proof of Proposition 1)

In this section we derive the first- and second-order unconditional moments of the liquidation count process Y_t under the stationarity conditions derived in the previous section.

i) Moments of order 1

From the conditional Poisson distribution of the counts, we have:

$$E_{t-1}(Y_t) \quad = \quad E_{t-1}[E_{t-1}(Y_t|F_t)] = E_{t-1}(a + bF_t + CY_{t-1}) = a + bE_{t-1}(F_t) + CY_{t-1},$$

where E_{t-1} denotes expectation conditional on the past histories of liquidation counts $\underline{Y_{t-1}}$ and factor $\underline{F_{t-1}}$. By taking the expectation of both sides of the equation, and using the stationarity of process (Y_t) and the normalization $E(F_t) = 1$, we get:

$$E(Y_t) = a + b + CE(Y_t) \quad \Leftrightarrow \quad E(Y_t) = (Id - C)^{-1}(a + b). \tag{a.11}$$

ii) Moments of order 2

Let us first consider the covariance between the liquidation counts and the frailty. We have:

$$\begin{aligned} E_{t-1}(F_t Y_t) &= E_{t-1}[E_{t-1}(F_t Y_t|F_t)] = E_{t-1}[F_t(a + bF_t + CY_{t-1})] \\ &= aE_{t-1}(F_t) + bE_{t-1}(F_t^2) + CE_{t-1}(F_t)Y_{t-1} \\ &= aE_{t-1}(F_t) + bE_{t-1}(F_t^2) + C(1 - \rho + \rho F_{t-1})Y_{t-1}, \end{aligned}$$

from equations (a.1) and (a.5). By taking the expectation of both sides of the equation, we get:

$$E(F_t Y_t) = a + b(1 + \sigma^2) + (1 - \rho)C(Id - C)^{-1}(a + b) + \rho CE(F_t Y_t),$$

where $\sigma^2 = V(F_t) = 1/\delta$ denotes the variance of the frailty [see Section A.1 iv)]. We deduce that:

$$\begin{aligned} E(F_t Y_t) &= (Id - \rho C)^{-1}\{b\sigma^2 + [Id + (1 - \rho)C(Id - C)^{-1}](a + b)\}, \\ &= (Id - \rho C)^{-1}b\sigma^2 + (Id - C)^{-1}(a + b). \end{aligned}$$

Thus:

$$Cov(Y_t, F_t) = \sigma^2(Id - \rho C)^{-1}b. \tag{a.12}$$

Let us now consider the variance-covariance matrix of the liquidation counts vector Y_t. We have:

$$\begin{aligned}
E_{t-1}(Y_tY_t') &= E_{t-1}(E_{t-1}[Y_tY_t'|F_t]) = E_{t-1}[V_{t-1}(Y_t|F_t) + E_{t-1}(Y_t|F_t)E_{t-1}(Y_t|F_t)'] \\
&= E_{t-1}[diag(a + bF_t + CY_{t-1})] + E_{t-1}[(a + bF_t + CY_{t-1})(a + bF_t + CY_{t-1})'] \\
&= diag[a + bE_{t-1}(F_t) + CY_{t-1}] + bb'V_{t-1}(F_t) \\
&\quad +[E_{t-1}(a + bF_t + CY_{t-1})][E_{t-1}(a + bF_t + CY_{t-1})]' \\
&= diag[a + bE_{t-1}(F_t) + CY_{t-1}] + bb'V_{t-1}(F_t) \\
&\quad +[a + b(1-\rho+\rho F_{t-1}) + CY_{t-1}][a + b(1-\rho+\rho F_{t-1}) + CY_{t-1}]'.
\end{aligned}$$

By taking the expectation of both sides, we deduce:

$$\begin{aligned}
E(Y_tY_t') &= diag[a + b + C(Id - C)^{-1}(a + b)] + bb'E[V_{t-1}(F_t)] + V[b\rho F_{t-1} + CY_{t-1}] \\
&\quad +E[a + b(1-\rho+\rho F_{t-1}) + CY_{t-1}]E[a + b(1-\rho+\rho F_{t-1}) + CY_{t-1}]' \\
&= diag[(Id - C)^{-1}(a + b)] + \sigma^2(1-\rho^2)bb' + V[b\rho F_{t-1} + CY_{t-1}] + E(Y_t)E(Y_t'),
\end{aligned}$$

where we use that $E[V_{t-1}(F_t)] = \sigma^2(1-\rho^2)$. Therefore, the variance-covariance matrix of Y_t satisfies the recursive equation:

$$\begin{aligned}
V(Y_t) &= CV(Y_t)C' + diag[(Id - C)^{-1}(a + b)] + \sigma^2 bb' \\
&\quad +\rho bCov(F_t, Y_t)C' + \rho CCov(Y_t, F_t)b'. \qquad \text{(a.13)}
\end{aligned}$$

Proposition 1 is obtained by substituting the expression (a.12) of $Cov(Y_t, F_t)$.

iii) Autocovariance at order 1

We have:

$$\begin{aligned}
Cov(Y_t, Y_{t-1}) &= Cov[E_{t-1}(Y_t), Y_{t-1}] = Cov[E_{t-1}(a + bF_t + CY_{t-1}), Y_{t-1}] \\
&= Cov[a + b(1-\rho+\rho F_{t-1}) + CY_{t-1}, Y_{t-1}].
\end{aligned}$$

Therefore:

$$\begin{aligned}
Cov(Y_t, Y_{t-1}) &= Cov(b\rho F_{t-1} + CY_{t-1}, Y_{t-1}) = b\rho Cov(F_{t-1}, Y_{t-1}) + CV(Y_t) \\
&= CV(Y_t) + \sigma^2\rho bb'(Id - \rho C')^{-1}. \qquad \text{(a.14)}
\end{aligned}$$

Appendix B: GMM estimation of the Poisson model with frailty and contagion

In this Appendix we develop an estimation approach for the parameters of the Poisson model with contagion and frailty that is based on the Generalized Method of Moments [GMM, Hansen (1982), Hansen, Singleton (1982)]. We start by considering moment restrictions that involve the liquidation intensity parameters and the parameters in the stationary distribution of the frailty only (Section B.1). We then present moment restrictions that involve the autoregressive parameter of the frailty as well (Section B.2). Finally, we discuss the implementation of the associated GMM estimators (Section B.3).

B.1 First-order moment restrictions

The moment restrictions are based on the special form of the conditional Laplace transform in equation (a.6). For management style k we have:

$$E[\exp(-u_k Y_{k,t}) | \underline{F_t}, \underline{Y_{t-1}}] = \exp\left\{-\gamma_{k,t}\left(a_k + b_k F_t + c_k' Y_{t-1}^*\right)(1 - e^{-u_k})\right\}, \tag{b.1}$$

for any argument $u_k \in [0, \infty)$. These conditional moments are appropriate for analyzing risk parameters. Indeed, the left hand side of the above equation is simply the expected utility function for an investor with a portfolio totally invested in the liquidation events of style k, and an absolute risk aversion equal to u_k. By considering the associated set of moment restrictions, we consider all types of investment, for all values of risk aversion. Therefore, the associated moment method will calibrate the unknown parameters on the whole set of expected utilities.

The equations in (b.1) can be rewritten as:

$$E\left[\exp\left\{-u_k Y_{k,t} + \gamma_{k,t}\left(a_k + c_k' Y_{t-1}^*\right)(1 - e^{-u_k})\right\} | \underline{F_t}, \underline{Y_{t-1}}\right] = \exp\left\{-\gamma_{k,t} b_k (1 - e^{-u_k}) F_t\right\}, \tag{b.2}$$

for $u_k \in [0, \infty)$. Thus, we obtain nonlinear transforms of the observable liquidation count variables, whose conditional expectation depends on the frailty only. As equation (b.2) holds for all real positive arguments u_k, we can consider a time-dependent argument u_k. The time dependence is selected such that the RHS of equation (b.2), and thus the LHS as well, is stationary. More precisely, let $u_{k,t}$ be such that $1 - e^{-u_{k,t}} = v/\gamma_{k,t}$, for given $v \in \mathcal{V}_k$, i.e. $u_{k,t} = -\log(1 - v/\gamma_{k,t})$. In order to obtain a well-defined $u_{k,t}$ in $[0, \infty)$, the real interval $\mathcal{V}_k$ has to be a subset of $[0, \inf_t \gamma_{k,t})$. Then, equation (b.2) becomes:

$$E\left[\exp\left\{\log(1 - v/\gamma_{k,t}) Y_{k,t} + v\left(a_k + c_k' Y_{t-1}^*\right)\right\} | \underline{F_t}, \underline{Y_{t-1}}\right] = \exp(-v b_k F_t), \quad \forall v \in \mathcal{V}_k. \tag{b.3}$$

These moment restrictions are conditional on factor path $\underline{F_t}$ and cannot be used directly for estimation since the factor is unobservable. Therefore, we integrate out the latent factor by taking expectation on both sides of the equation w.r.t. the gamma stationary distribution $\gamma(\delta, 0, 1/\delta)$ of factor process F_t [see Appendix A.1 iv)]. We get a continuum of unconditional moment restrictions:

$$E\left[\exp\left\{\log(1 - v/\gamma_{k,t}) Y_{k,t} + v\left(a_k + c_k' Y_{t-1}^*\right)\right\}\right] = \frac{1}{(1 + v b_k/\delta)^{\delta}}, \quad \forall v \in \mathcal{V}_k. \tag{b.4}$$

These first-order moment restrictions involve the intensity parameters a_k, b_k, c_k for any type k, as well as parameter δ characterizing the stationary distribution of the frailty, but do not allow to identify the frailty persistence parameter ρ.

B.2 Second-order moment restrictions

In order to derive moment restrictions that allow for estimation of parameter ρ, let us consider equation (b.3) and multiply both sides by $\exp(-\tilde{u}_{l,t-1}Y_{l,t-1})$, where $\tilde{u}_{l,t-1} = -\log(1-\tilde{v}/\gamma_{l,t-1})$, for some type l and any $\tilde{v} \in \mathcal{V}_l$. We have:

$$E\left[\exp\left\{-u_{k,t}Y_{k,t} - \tilde{u}_{l,t-1}Y_{l,t-1} + v\left(a_k + c_k' Y_{t-1}^*\right)\right\} | \underline{F_t}, \underline{Y_{t-1}}\right] = \exp\left(-vb_k F_t - \tilde{u}_{l,t-1}Y_{l,t-1}\right),$$

for all $v \in \mathcal{V}_k, \tilde{v} \in \mathcal{V}_l$, where $u_{k,t} = \log(1 - v/\gamma_{k,t})$. By taking the conditional expectation given $\underline{F_t}$ and the liquidation counts history $\underline{Y_{t-2}}$ up to month $t-2$ on both sides of the equation, we get:

$$\begin{aligned} & E\left[\exp\left\{-u_{k,t}Y_{k,t} - \tilde{u}_{l,t-1}Y_{l,t-1} + v\left(a_k + c_k' Y_{t-1}^*\right)\right\} | \underline{F_t}, \underline{Y_{t-2}}\right] \\ = \; & \exp\left(-vb_k F_t\right) E\left[\exp\left(-\tilde{u}_{l,t-1}Y_{l,t-1}\right) | \underline{F_t}, \underline{Y_{t-2}}\right] = \exp\left\{-vb_k F_t - (a_l + b_l F_{t-1} + c_l' Y_{t-2}^*)\tilde{v}\right\}. \end{aligned}$$

By rearranging terms, and computing the unconditional expectation of both sides, we get:

$$E\left[\exp\left\{-u_{k,t}Y_{k,t} - \tilde{u}_{l,t-1}Y_{l,t-1} + v\left(a_k + c_k' Y_{t-1}^*\right) + \tilde{v}\left(a_l + c_l' Y_{t-2}^*\right)\right\}\right] = E\left[\exp\left(-vb_k F_t - \tilde{v} b_l F_{t-1}\right)\right],$$

for all $v \in \mathcal{V}_k, \tilde{v} \in \mathcal{V}_l$. The expectation in the right-hand side involves the joint distribution of the frailty values F_t and F_{t-1} on two consecutive months, and hence depends on the frailty autocorrelation parameter ρ. In fact, by the exponential affine property of the ARG process (see Appendix A.1) we have:

$$\begin{aligned} & E\left[\exp\left(-vb_k F_t - \tilde{v} b_l F_{t-1}\right)\right] \\ = \; & E\left[E[\exp\left(-vb_k F_t\right) | F_{t-1}] \exp\left(-\tilde{v} b_l F_{t-1}\right)\right] = \exp\left(-\beta(vb_k)\right) E\left[\exp\left(-[\alpha(vb_k) + \tilde{v} b_l] F_{t-1}\right)\right] \\ = \; & \frac{1}{[1 + (1-\rho)vb_k/\delta]^\delta} \frac{1}{[1 + [\alpha(bv_k) + \tilde{v} b_l]/\delta]^\delta} = \frac{1}{[1 + (vb_k + \tilde{v} b_l)/\delta + (1-\rho)v\tilde{v} b_k b_l/\delta^2]^\delta}, \end{aligned}$$

where we use $\beta(u) = \delta \log(1 + (1-\rho)u/\delta)$ and $\alpha(u) = \dfrac{\rho u}{1 + (1-\rho)u/\delta}$ from equations (a.3) and (a.5). Thus, we get the continuum of unconditional dynamic moment restrictions:

$$\begin{aligned} & E\left[\exp\left\{\log(1 - v/\gamma_{k,t})Y_{k,t} + \log(1 - \tilde{v}/\gamma_{l,t-1})Y_{l,t-1} + v\left(a_k + c_k' Y_{t-1}^*\right) + \tilde{v}\left(a_l + c_l' Y_{t-2}^*\right)\right\}\right] \\ = \; & \frac{1}{[1 + (vb_k + \tilde{v} b_l)/\delta + (1-\rho)v\tilde{v} b_k b_l/\delta^2]^\delta}, \quad \forall v \in \mathcal{V}_k, \tilde{v} \in \mathcal{V}_l. \end{aligned} \tag{b.5}$$

These second-order moment restrictions, written for all pairs of management styles (k, l), involve all model parameters. For $\tilde{v} = 0$, these moment restrictions reduce to the static moment restrictions (b.4).

B.3 GMM estimators

The moment restrictions in Sections B.1 and B.2 can be used in different ways to define GMM estimators. We consider below three possible GMM approaches.

i) We can use the first-order moment restrictions (b.4) written for all types k to define a first-step GMM estimator of parameters a_k, b_k, c_k, for all management styles k, and parameter δ. Then, a second-step GMM estimator of parameter ρ can be defined by using some subset of the second-order moment restrictions (b.5) and replacing parameters a_k, b_k, c_k and δ with their first-step GMM estimates.

ii) Alternatively, the second-order moment restrictions (b.5) can be used to estimate jointly all model parameters $(\theta', \varphi')'$, where $\theta = (a_k, b_k, c_k', k = 1, ..., K)'$ and $\varphi = (\delta, \rho)'$, in one step. More precisely, consider the second-order moment restrictions (b.5) written for all management styles k, with $l = k$, and for a given grid of values $v, \tilde{v}$. [17] They define a set of unconditional moment restrictions $E\left[h_{k,t}(a_k, b_k, c_k, \varphi)\right] = 0$, for $k = 1, ..., K$, say, where the $h_{k,t}$ are the moment functions. Then, the GMM estimator $(\hat{\theta}', \hat{\varphi}')'$ minimizes the criterion:

$$Q_T(\theta, \varphi) = \sum_{k=1}^{K} \|\hat{h}_{k,T}(a_k, b_k, c_k, \varphi)\|^2, \tag{b.6}$$

where $\hat{h}_{k,T}(a_k, b_k, c_k, \varphi) = \frac{1}{T}\sum_{t=1}^{T} h_{k,t}(a_k, b_k, c_k, \varphi)$ is the sample average of the orthogonality function. By including only the moment restrictions with $k = l$, and using a weighting matrix that is diagonal across management styles, we simplify considerably the optimization problem. [18] In fact, for given value of the bivariate frailty parameter $\varphi = (\delta, \rho)'$, the GMM criterion (b.6) is additive w.r.t. the parameters of the styles. Therefore, the minimization can be performed by concentrating w.r.t. parameter θ. Namely, for given φ, the criteria for the different styles k are separately minimized w.r.t. parameters a_k, b_k, c_k by a Newton-Raphson algorithm. Then, the concentrated criterion is minimized by a bi-dimensional grid search over parameters (δ, ρ).

iii) We can also consider the GMM estimators that exploit the full set of second-order moment restrictions (b.5) written for all pairs of management styles (k, l). Such GMM estimators are expected to be more efficient than the GMM estimator minimizing criterion (b.6). However, such GMM estimators are computationally less convenient, because they involve an optimization over a large-dimensional parameter space that can not be dealt with by concentration easily.

In this paper we apply method *ii)*, which provides a trade-off between tractability and efficiency.

[17] It might also be possible to apply an efficient GMM taking into account the continuum of moment restrictions (b.5) [see e.g. Carrasco et al (2007)].

[18] For expository purpose the weighting matrix is set equal to the identity matrix for each type.