MVA, KVA: modelling challenges

Moez MRAD
Head of Credit & XVA Quantitative Research
moez.mrad@ca-cib.com

Views and opinions expressed in this presentation are the personal ones of the speaker. They may differ from current, past or future employer views, policies and interpretations.
Some CCR Challenges / Expected XVA

- **Fundamental Review of the Trading Book (FRTB) - 2018:**
  - CVA VaR calculations for IR and FX will require the computation of several sensitivities on EPE profiles.
  - Failure to meet the requirements for the FTRB approach will result in a fall back to the standardized approach (SA-CVA) and higher RWA/Capital costs as a result.

- **Initial margin requirements (MVA) – 2016:**
  - BCBS: “All financial firms that engage in non-centrally cleared derivatives must exchange initial and variation margin as appropriate to the counterparty risks posed by such transactions.”
  - The standardized approach is add-on based / may be replaced by SA-CCR later on.
  - Failure to come up with an internal model will result in a fall back to the standardized approach.

- **Capital consumption:**
  - It is important to estimate the KVA: the present value of the future capital costs due to the various charges
  - Internal Model: To get the term structure of UL we have to nest a VaR like computation (the UL at the desired confidence level and time horizon) within the Monte Carlo simulation.
Initial Margin and MVA

- IM posted has to be funded, IM received cannot be re-hypothecated.
- Margin Value Adjustment (MVA): Expected future funding cost due to IM.
- IM reduces over-collateralization which lead to less CVA.
- IM also reduces the need for capital as it reduces counterparty credit risk.
- Credit risk turned into liquidity risk and credit-related costs into funding costs.

Internal Model based method

- Calibrated for 99% confidence interval, Margin Period of Risk at least 10 days.
- Compute IM per netting set and asset class.
- Sophisticated enough to cope with regulatory requirements.
- **Brute force approach**: Bump risk factors and revalue. It needs a high computational effort. Can not be achieved without use of specific technology (GPU and/or other recent technologies).

- **Numerical techniques based approach**:
  - Compute analytic derivatives (sensitivities) per trade. Possible for the trades eligible for CCPs.
  - Use sensitivities with historic market moves to compute the exposure distribution.
  - Derive VaR or Expected Shortfall directly from the distribution (similar to parametric VaR).
  - More complex products can be handled in a similar way (semi-exotics) or replaced by a regression.
Main sources of capital consumption for derivatives:

- Counterparty Credit Risk (CCR).
- CVA risk charge.
- Market risk (VaR and Stressed VaR).

Capital consumption depends on:

- Counterparty nature: CCPs and many non-financials are exempt from CVA charge and/or have lower CCR.
- CSA (via CVA risk charge and credit exposure):
  - high volume, complexity or disputes increase MPR and hence CVA risk charge.
  - collateralization and initial margins reduce CCR charge.
- Regulator and jurisdiction – Exemptions from CVA risk charge vary in different jurisdictions
- Availability of CDS protection to reduces CVA risk charge
- Received Initial Margin (and/or independent amount)
- Diffusion Models, measures, calibration…
Why is capital required on top of the other XVA (Credit risk, Funding cost…) ?

- Other XVA adjust accounting for the expected loss (or expected cost) over the life of a transaction as seen at \( t=0 \).
- Unexpected loss arise as well. They do not impact accounting as not a fair value (a drift) but rather a risk.
- These unexpected losses have to be covered by allocated Bank’s capital, for an amount corresponding to the desired confidence level the Bank wants being able absorbing these unexpected losses at.

Capital value Adjustment (KVA): Up fronted value of unexpected losses

How to upfront unexpected losses?

- Up front revenues over the whole life of the transactions / portfolio.
- Getting the initial unexpected loss, even if not straightforward, isn’t sufficient
- What is required is the whole forward profile of the time and market dependent term structure of UL…
- A calculation difficulty arises here
  - To get the term structure of UL means we have to nest a VaR like computation (The UL at the desired confidence level and time horizon) within the Monte Carlo simulation getting us the EL in the first place.
  - If the VaR to be computation relies itself on a numerical procedure (MC or historical VaR), we multiply in a potentially explosive way the number of valuations to be carried
  - Brute force approach or smart numerical methods might be considered when applicable
Numerical approaches for sensitivities computation

**AAD**

- XVA are functions with lots of input variables (such as market data).
- Sensitivities are approximations of the mathematical derivatives.
- Derivatives can be computed via:
  - Finite differences (FD).
  - Adjoint Algorithmic Differentiation (AAD).
- AAD can be implemented via:
  - Operator overloading (advised for XVA).
  - Source transformation.
- All the sensitivities will be displayed in one single sweep: same computation time for 10 or 1000 sensitivities!
## AAD Pros and Cons

<table>
<thead>
<tr>
<th>Pros</th>
<th>Cons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensitivity to n inputs is computed in constant time!</td>
<td>Memory consuming!</td>
</tr>
<tr>
<td>Well suited to Monte-Carlo simulations</td>
<td>Implementation of ADD is quite tricky in practice</td>
</tr>
<tr>
<td>Multithreading/parallel computing compatibility</td>
<td>Does not improve (significantly) sensi precision</td>
</tr>
<tr>
<td>C++ compliant</td>
<td>Not as intuitive as FD</td>
</tr>
<tr>
<td>Robustness</td>
<td></td>
</tr>
<tr>
<td>Efficient for processes with high computation cost like XVAs estimation.</td>
<td></td>
</tr>
</tbody>
</table>
AAD methodology can be divided into three fundamental steps:

1. **Tree construction**: the XVA estimation can be seen as a sequence of basic operations i.e. a pricing tree.
2. **Tree evaluation**: the pricing tree is evaluated for each simulation scenario.
3. **Adjoint Backward propagation**: sensitivities are computed in a backward mode: the tree is browsed from output (XVA) to inputs (market data).

Thus, it is necessary to store:

- All the operators of the pricing tree (done in step 1).
- Results of these operators for each Monte Carlo Simulation (step 2).

All the computation time for the three steps should not exceed 4 times the standard pricing computation time in theory. In practice, closer to 8.
800 sensitivities in AAD performance in CVA single evaluation units

- Construction: 0.936
- Evaluation: 0.893
- AAD propagation: 3.269
SIMM Extended to MVA

- Compute sensitivities for each scenarios and each time step (analytically and/or via AAD)
- Use of sensitivities in order to get valuation changes:
  1. First choice consider historical VaR shocks;
     - and use sensitivities to estimation portfolio valuation changes;
     - then sort scenarios and compute the 99% quantile.
  2. Second choice is to use parametric VaR techniques:
     - compute the square root of the sum of squared portfolio valuation change,
     - compute the root mean square
     - multiply it by a normal standard deviate corresponding to 99% and derive a normal or Gaussian covariance VaR approach.
MVA by replication

Compute a VaR inside a Monte carlo simulation:

1. Use Longstaff-Schwartz regression function for all products of the portfolio.

2. Longstaff-Schwartz in its original form require “Augmentation” in order to estimate VaR efficiently. Two augmentation methods:
   - **Early State Augmentation (Wang & Calfish (2009))**:
     - Simulations start earlier than today.
     - Preserve path continuity (needed for portfolio with bermudan products).
   - **Shocked State Augmentation (Green & Kenyon (2014))**:
     - Apply one VaR shock at each simulation date, on each path.
     - Simpler, parsimonious, does not require extra simulation.
     - Only for non bermudan products (example cleared ones).

Apply shocks and compute VaR for each time and each scenario.
KVA computation (I)

- KVA is up fronting future capital consumption $K(t)$, $0 < t < T$.

\[ KVA = -\int_0^T h(t)B(0, t)p_{c,B}(t)E^Q[K(t)]dt \]

- $K(t) = K_{MR}(t) + K_{CVA}(t) + K_{CCR}(t)$

  \[ KVA = KVA_{MR} + KVA_{CVA} + KVA_{CCR} \]

- $KVA_{MR}$ can be computed using
  - standardized Approach (Basle II): $K_{MR}$ is not dependent on market conditions.
  - Internal Model: Nested VaR computation (Similar to MVA)

- $KVA_{CVA}$ can be computed using
  - standardized Approach (Basel III): We end up by up-fronting EAD (t).
  - Internal Model: Nested CVA Credit VaR computation (Similar to MVA)
KVA computation (II)

$K_{VA_{CCR}}$ can be computed by up fronting $K_{CCR}(t) = weight \times 12.5\% \times EAD(t)$

- Weight is computed via: (1) standardized or (II) Foundation or Advanced IRB approaches.

- $EAD(t)$ can be computed as in:
  
  1. Current Exposure Method: MtM + add-on / some netting is allowed for add-on.
  2. standardized approach: formula depending on transaction, collateral and their risk.
  3. SA-CCR: tries to mimic the IMM approach but still depend on MtM at simulation time.

$\Rightarrow$ KVA computation complexity similar to CVA or DVA (especially in CEM).

4. Internal model:
   - Exposure at default $EAD(t) = 1.4 \times EEPE(t)$
   - Effective expected positive exposure
     
     $EEPE(t) = \frac{1}{(T-t)\wedge 1} \int_0^{(T-t)\wedge 1} \max_{0<u<v} EE(t+u)dv$

   - Each time and scenario we need to estimate EEPE (nested Monte Carlo !)
   - In practice solved by American Monte Carlo techniques or Branching Process (see Andreasen (2015))